

**A Comparison of the Intended Mathematics Curriculum in
China, Hong Kong and England
and the Implementation in
Beijing, Hong Kong and London**

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Abstract

This thesis compares the various components of the intended junior secondary mathematics curriculum in China, Hong Kong and England and the implementation in Beijing, Hong Kong and London, and suggests factors to account for the differences in curriculum found.

The three places under study were found to differ tremendously in their culture, education system and history of development of the mathematics curriculum. For the intended curriculum, the three places did not differ much in the aims of mathematics education on the utility of the product of mathematics, but great differences were found in the intrinsic aims and aims on the utility of the process of mathematics. The curriculum in all three places intended the use of a variety of teaching methods, but great differences were found in the mathematics content intended to be learned by the students. Results of a questionnaire showed that teachers in the three places differed greatly in their attitudes towards mathematics and mathematics education. Moreover, a pattern of responses emerged, with the responses of the Hong Kong teachers lying between those of the Beijing and London teachers.

Through classroom observations, it was found that teachers in the three places differed in their classroom teaching, and analysis of selected topics from textbooks revealed that students in the three places learned very different mathematics in the classroom. Despite these differences, mismatch between the intended and implemented curriculum was found in all three places, although the kind of mismatch differed from place to place.

Lastly, a framework of factors at four levels was suggested to account for the differences in curriculum. It was argued that factors at the classroom, school and societal levels all contributed to the differences found, but factors at the cultural level had to be drawn upon for a satisfactory explanation of the findings in this study.

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Frederick K.S. Leung

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To Shirley and King Ho

Table of Contents

	page
List of Tables and Figures	8
Chapter 1 Introduction	10
Delimitations	13
The Research Problem	17
Chapter 2 Literature Review	20
Curriculum	21
Past Studies	33
Chapter 3 Methodology	54
Data Collection	54
The Preliminary Visits	60
Development of the Instruments for Data Collection	65
The Pilot Study	77
The Main Study	82
Chapter 4 The Contexts of the Curriculum	89
The Chinese Culture in Contrast with Western Cultures	90
The Education System in China, England and Hong Kong	98
A Brief History of Development of the Mathematics Curriculum in China, England and Hong Kong	108

Chapter 5	The Intended Curriculum	128
	Aims and Objectives of Mathematics Education for Junior Secondary School Students	131
	Intended Teaching Methods	138
	The Intended Mathematics Content	143
Chapter 6	Teachers' Attitudes towards Mathematics and Mathematics Teaching and Learning	150
	Characteristics of the Teachers Who Responded to the Questionnaire	150
	Teachers' Views of Mathematics - the Mathematics as a Process Scale	153
	Teachers' Views on Mathematics Education	158
	Summary	175
Chapter 7	The Implemented Curriculum	177
	Method of Data Analysis	178
	Results of the Data Analysis	184
	Summary	215
Chapter 8	Content of the Implemented Curriculum: Analysis of Selected Topics from the Textbooks in the Three Places	217
	Introduction	217
	Negative Numbers	220
	Quadratic Equations and Quadratic Functions	231
	Isosceles Triangles	238
	Statistics	247
	Summary	256

Chapter 9	Overall Discussion	258
	Consistency of the Intended Curriculum and Implemented Curriculum	259
	A Framework for Explaining Differences in Curriculum	266
	Explanations of Differences in the Mathematics Curriculum	269
Chapter 10	Conclusion	281
	Summary of Major Findings	281
	Implications of the Findings	288
	Limitations	292
	Further Research	295
	Concluding Remarks	297
	BIBLIOGRAPHY	298
	APPENDICES	
A	List of People Interviewed	315
B	Questionnaire for the Pilot Study	317
C	Questionnaire for the Main Study	322
	I. Beijing	323
	II. Hong Kong	325
	III. London	327
D	Classroom Observation Schedule for the Pilot Study	329
E	Classroom Observation Schedule for the Main Study	330
F	Interview Schedule (for Teachers)	331
G	Juxtaposition of the Content in the three Curricula	332
H	Subtraction of Negative Numbers	339

List of Tables and Figures

Tables

3.1	Number of Lessons Observed and Copies of Questionnaire Received in the Pilot Study	78
3.2	Distribution of Grades of Lessons Observed in the Main Study	86
5.1	Aims on the Utility of the Product of Mathematics	134
5.2	Aims on the Utility of the Process of Mathematics	135
5.3	Intrinsic Aims	135
5.4	Intended Teaching Methods	141
5.5	Percentage of Time Spent on Various Mathematics Topics	144
5.6	A Simplified Chart Showing the Mathematics Content	145
6.1	Characteristics of Teachers Who Responded to the Questionnaire	150
6.2	The Mathematics as a Process Scale	155
6.3	Teachers' Views on the Most Important School Subjects	159
6.4	Teachers' Views on the Aims of Mathematics Education	160
6.5	Influences on Mathematics Content Taught in the Classroom	164
6.6	Factors Affecting Students' Success or Failure	166
6.7	Amount of Homework Expected of Students	168
6.8	Teachers' Attitudes Towards Gender Differences	169
6.9	Means Table for Item 1 of Section C	170
6.10	Results for Item 2 of Section C	171
6.11	Results for Item 3 of Section C	172
6.12	Results for Item 4 of Section C	173

7.1	Number of Students in a Class	177
7.2	Durations of Various Events in the Classroom	185
7.3	Occurrences of Events Related to Classroom Practice	186
7.4	Events Related to the Intended Curriculum	198
7.5	Events Related to Views of Mathematics	201
7.6	Events Related to Views of Mathematics Teaching and Learning	211

Figures

2.1	Tyler's Model	22
2.2	Wheeler's Model	23
2.3	Kerr's Model	24
2.4	Taylor's Model	25
2.5	The SIMS Model	27
2.6	A Two-way Grid: Tyler's Model x SIMS Model	29
2.7	An Improved Grid: Tyler's Model x SIMS Model	29
2.8	Model Used in the Present Study	30
3.1	Methods of Data Collection in this Study	59
5.1	Relative Emphases on Content	147
6.1	Graph of the Means Table for Item 1 of Section C	170
7.1	Flow of Activities in a Typical Beijing Lesson	188
7.2	Flow of Activities in a Typical Hong Kong Lesson	189
7.3	Flow of Activities in a Typical London Lesson (Individualized Programme)	190
7.4	Flow of Activities in a Typical London Lesson (Whole-class Teaching)	191

Chapter 1: Introduction

"If education is regarded as one of the tools for national advancement", says Eckstein *et al* (1982:5), "then curriculum lays out part of the plan and serves as the means for achieving it. As such, it warrants systematic study." Within the overall curriculum in schools, the mother tongue language and mathematics have always occupied central places, showing that these two subjects are in general considered essential for children. In particular, mathematics is often thought to be of special importance among school subjects (see for example Cockcroft, 1982:paragraph 2).

One useful way of studying the mathematics curriculum is through comparison with the curriculum in another country or system. It is through comparative studies that the strengths of a curriculum are recognised and its weaknesses exposed. Mathematics is often considered a good school subject for comparative studies because it is considered a "universal" subject, relying heavily on logic and symbolic notation and is thus relatively language-independent, and therefore simplest for the purpose of comparison. However, Stigler and Perry (1988a:220-221), in reporting on a series of studies directed by Stevenson at the University of Michigan (see chapter two of this thesis), expressed amazement at the "small number, and relatively narrow scope, of cross-cultural studies (in mathematics education) that have been done", and they suggested that "what have been particularly lacking ... are studies of how mathematics is taught in classrooms in different cultures". This study attempts to investigate the mathematics curriculum in the People's Republic of China (hereafter referred to as China), Hong Kong and England both at the

intended level and the classroom teaching level, supplying part of that information which Stigler and Perry found so lacking.

A comparison of the curriculum in these three places has a special significance for Hong Kong. With the signing of the joint declaration by Britain and China in December 1984, the British Colony entered a transition period of 13 years before its sovereignty is returned to China. One of the decisions that educators in Hong Kong are facing is the kind of changes that need to be made in the education system in general and in the curriculum in particular to achieve a smooth transfer of sovereignty. A comparative study in curriculum will provide useful information for such decision making.

Hong Kong has been a British colony for more than one and a half centuries, and as a colony, every aspect of the community, including the important component of education, is affected by the ruling state. However, because of her origin from and proximity with China, Hong Kong has never lost her cultural link with the motherland. On the contrary, being the most homogeneously ethnic Chinese community outside Mainland China and Taiwan, most of the Chinese traditions and values are still retained. Hong Kong's dramatic prosperity under the British rule and the consequent development into one of the world's major communication and financial centres has however made it a city prone to influence by Western cultures. It is this particular cultural location of Hong Kong with respect to both China and England that makes the present study most significant. It is believed that a comparative study of this China-Hong Kong-England trio will enable us to draw on cultural factors to help interpret the results of the comparative curriculum study.

The Chinese account for more than one fifth of the world's population. The Chinese culture is one of the major cultures in the world, and is the dominant culture in the countries of East Asia. Westerners have always been interested in the culture of China, and recently, research interests in the education and achievement of pupils of Chinese and East Asian origin have grown substantially, especially in the United States (see for example Becker *et al*, 1990; Chen, 1991; Chung and Walkey, 1989; Shimahara, 1986; Stigler and Stevenson, 1991; Yan and Eugene, 1991). Research into the education of pupils in China itself, however, has been very limited because of the political situation in the country. Before the end of the Cultural Revolution in 1976, this Communist regime was basically closed to outsiders as far as educational research is concerned. Even after 1978 when China started to adopt an "open policy", it has still been very reluctant to allow foreign researchers to conduct educational research in its territory. A comparative study that involves China will help fill in part of this gap of knowledge that exists in educational research.

What follows in this chapter are delimitations of the scope of the study. Then the research problem of the study will be stated, and an outline of the rest of the chapters in this thesis will be given.

Delimitations

CHINA

In this thesis, China refers to the People's Republic of China. It is where the vast majority of people of Chinese origin are found and yet, as pointed out above, one of the countries where the curriculum is least studied. China is a vast country with a huge population. With regard to the implementation of the mathematics curriculum, it has not been feasible in this study even to sample from the 30 provinces and regions of the country with their diverse geographic and economic backgrounds. Therefore, it was decided that only the capital city of Beijing would be used to illustrate the implemented curriculum.

ENGLAND

England is part of the United Kingdom, which comprises England, Wales, Scotland and Northern Ireland. To conduct a study that takes into account the differences between the four regions of the Kingdom would be too complicated, and does not seem to contribute greatly to the focus of the present study. Narrowing down the study to include only England has the advantage that the problem of languages (such as teaching in the medium of Welsh) within the Kingdom does not need to be addressed. Again, like the case in China, the capital city of London was chosen for studying the implemented curriculum.

HONG KONG

Hong Kong is a metropolitan city comparable to Beijing and London in terms of its area and population. It is a homogeneously Chinese community with 98% of its population ethnically Chinese. There are a number of international schools that cater for the children of expatriates, but these have been excluded in this study.

CURRICULUM

A crucial concept used in this study is that of a curriculum. A clear definition of this concept is extremely important because it determines exactly what we are comparing. "Just to describe a curriculum is a complex task" (Jenkins and Shipman, 1976:6), and the concept of a curriculum will be defined in chapter two after the relevant literature is reviewed. In the definition given in chapter two, a curriculum includes the end-product of the educational programme (attained curriculum). But because of practical difficulties in administering tests to students in China, and in order to limit the scale of the present study, this component of the curriculum has not been studied.

It is also necessary for practical reasons to select and limit the scope of the other components of the curriculum in this study. In all three places, only students studying in the mainstream have been included for study. So for example, students with special needs, evening schools that cater for adults etc. do not fall into the scope of the present study. Also, the junior or lower secondary level has been chosen as the level under study.

It refers to the first three years of secondary education or the 7th to 9th years of formal education in each of the three places. In China and Hong Kong, this means looking into the curriculum for those children who are between 12 and 15 years old, while in England it is for those between 11 and 14 (because students enter primary one at the age of 6 in China and Hong Kong and at the age of 5 in England).

The junior secondary level is an important stage in a child's education. It is the transition between what the UNESCO (1990:2-1) International Standard Classification of Education (ISCED) defines as First Level and Second Level schooling, and "it is here that many features of the school experience of students change (classroom organization, selection into school types and ability groups, etc.)" (Travers and Westbury, 1989:58). In mathematics, this stage is particularly significant because usually it represents where "elementary" mathematics ends and the first course in algebra begins.

For China and Hong Kong, there is a further significance in choosing this stage for study. For the pupils in these two places, the junior secondary school is the last stage of their compulsory education. As far as intentions are concerned, the curriculum at this stage represents the minimum of what the curriculum developers expect every citizen to attain. In England, although compulsory education extends to the age of 16, the first three years of secondary school are usually also regarded as time for laying a basic foundation for all, while there is more differentiation in the 4th and 5th years when students prepare more intensely for the differentiated GCSE examinations.

CULTURE

As pointed out above, the present study intends to draw on cultural factors in interpreting the results of the comparison of the curriculum in the three places. Culture is "one of the two or three most complicated words in the English language" (Williams, 1976:76). It can be defined as "the fabric of ideas, ideals, beliefs, skills, tools, aesthetic objects, methods of thinking, customs and institutions" (Smith, Stanley and Shores, 1957:4) or the "*configuration or generalization of the 'spirit' which informed the 'whole way of life' of a distinct people*" (Williams, 1981:10), and Reynolds and Skilbeck (1976:30) regard culture as "the field of interaction between (1) men's social relationships and conventions, (2) the symbolic forms available to them for focusing on and coordinating experience, and (3) their system of belief, values and action." In this thesis culture is taken to mean shared values, attitudes, beliefs and practices, roughly corresponds to the third component in Reynolds and Skilbeck's definition.

Hofstede (1980:25-26) treats culture as "the collective programming of the mind which distinguishes the members of one human group from another" and reserves the use of the word for societies (or nations) or for ethnic and regional groups. Within a smaller group in society, shared values, attitudes, beliefs and practices may also develop, but the term "sub-culture" is usually used to describe these. Sub-cultures are related to the culture of the society in which they are found, and are constrained by the general culture. In this thesis, the sub-cultures of interest are the professional sub-culture of teachers in general, and of mathematics teachers in particular.

The Research Problem

The key research problem of the present study is:

What are the differences in the various components of the junior secondary mathematics curricula between China, Hong Kong and England, and what factors can be suggested to account for these differences?

This key problem can be broken down into the following questions:

1. Can a model of a curriculum be developed based on which the mathematics curricula in the three places can be compared?
2. What are the contexts that give rise to the current mathematics curricula in the three places?

Intended Curriculum

3. What are the differences in aims and objectives of mathematics education in the three places?
4. What are the differences in intended teaching methods in the three places?
5. What are the differences in intended mathematics content in the three places?

Teachers' Attitudes

6. What are the differences in teachers' attitudes towards mathematics and mathematics education in the three places?

Implemented Curriculum

7. What are the differences in methods of teaching in the three places?
8. What are the differences in the mathematics content taught in the three places?

Interpretation

9. Are the various components of the curricula in the three places consistent with each other?
10. What factors can be suggested to account for the differences in curricula in the three places?

These questions will be discussed in turn in the chapters that follow:

In chapter two, the relevant literature will be reviewed in order to develop a curriculum model for use in this study. Past comparative studies in mathematics curriculum, especially when they involved any one or more of the three places, will also be reviewed so that the present study can benefit from the work of others.

In chapter three, the methodology for the present study will be presented. The kind of data collected and the development of the instruments for data collection will be discussed, and the ways the pilot study and the main study were conducted will be described.

Chapter four is meant to set the context for the comparison. The relevant literature on the cultural differences between the three places will be reviewed, and the education system and history of development of the mathematics curriculum in the three places will be briefly described.

The results of the main study will be presented in chapters five to eight. The aims and objectives of mathematics education, the intended teaching methods, the intended mathematics content, results of a questionnaire on teachers' attitudes, classroom observations on the implemented curricula, and selected topics from mathematics textbooks will in turn be compared and discussed.

Chapter nine attempts to assess the consistency of the intended and implemented curriculum in the three places respectively. Then a framework will be suggested to explain the differences in curriculum found in this study.

Chapter ten will summarize the major findings of the present study, suggest some implications of the findings, point out the limitations of the study and point to some possible further research.

Chapter 2: Literature Review

Chapter one outlined the research problem of the present study. Before discussing the methodology to be used for the study in chapter three, there is a need to delineate what is meant by the term "curriculum" used in this thesis. It is also important to be fully aware of relevant research on comparative studies in mathematics curriculum done in the past, so that the present study can benefit from these past studies and build upon the knowledge accumulated so far.

The best way to achieve the above is to review the literature concerning the different conceptualizations and usage of the term "curriculum", and derive from the review a model of the curriculum that suits the needs of the present study. Then relevant comparative studies on curriculum will be reviewed and discussed, their contributions acknowledged and their limitations pointed out. The implications of these past studies for the present study will be summarized and taken note of in the design of the present study.

Curriculum

According to Taba (1962:3), the foundations for a comprehensive theory of curriculum planning were only laid in the 1930's, although studies related to the curriculum have a much longer history. Before the 1930's, educators had very vague notions about the term "curriculum". It was roughly used to refer to the course of study offered by an educational institution¹, and by a course of study was usually meant the content of the instruction. In fact, many educators at that time did not distinguish between a curriculum and a syllabus. Even relatively recently, some educators have still used this view of a curriculum. The mathematics educator Begle, for example, considered a mathematics curriculum as "mathematical objects ... and the content of the instructional program devoted to these mathematical objects" (Begle, 1979:59).

However, Begle is by no means typical among recent or current educators in the usage of the term curriculum. Any current textbook on curriculum studies would give a definition encompassing more than the mere content for instruction. Different educators have different conceptualizations of a curriculum. As far as the present study is concerned, what is needed is a comprehensive model specifying the components of a curriculum so as to guide the data collection and analysis exercise.

In the early post-War period, Ralph Tyler was perhaps the first educator to systematically study this concept of curriculum and insisted that it was more than mere content plus

¹ the Oxford Advanced Learner's Dictionary renders the meaning of the term curriculum as "course of study in a school, college, etc."

instruction. In his classic book "Basic Principles of Curriculum and Instruction", he listed four basic questions to be considered in connection with a curriculum (1949:1). The four questions are:

1. What educational purposes should the school seek to attain?
2. What educational experiences can be provided that are likely to attain these purposes?
3. How can these educational experiences be effectively organised?
4. How can we determine whether these purposes are being attained?

From the time of Tyler, educators began to talk about "curriculum models", and Tyler's model was conceived as a linear one, consisting of four components, as shown below:

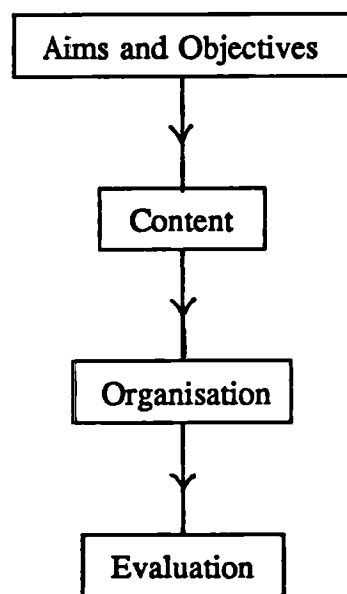


fig. 2.1: Tyler's Model

The model specifies not only the components of a curriculum, but also the relationship between the components. In figure 2.1, the arrows denote logical (and hence

chronological) relationships. Hence according to this model, we should first of all determine the aims and objectives of the educational programme. Based on the aims and objectives, the content of instruction is chosen, which in turn determines how the content is to be organised. Lastly, to complete the process, clear specification of how the aims and objectives are to be evaluated has to be laid down. It should be pointed out here that the components in the model are designed to answer the four questions listed above, and so "evaluation" here refers to pupil assessment in relation to the curriculum design rather than evaluation of the curriculum itself.

Most curriculum models can be considered derivatives of Tyler's model. Wheeler (1967:31), for example, suggested a cyclical model with components nearly identical to those in the Tyler's model, as shown in the diagram below:

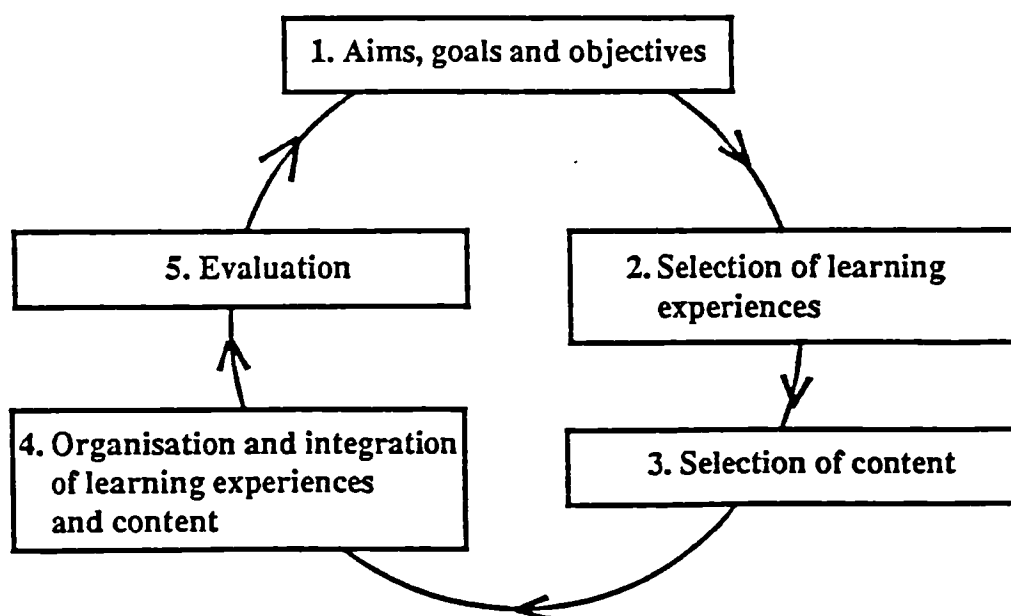


fig. 2.2: Wheeler's Model

From figure 2.2, it can be seen that if the second and third phases of Wheeler's model were combined, the phases in the model reduce to the four components in Tyler's model. The contribution of Wheeler, however, is his stress on the recycling nature of the curriculum development process (which is in fact implicit in Tyler's original model), as indicated by the cyclical shape of the model. Hence the curriculum is seen not as a static structure but as the product of an ongoing, developmental exercise. The evaluation phase (i.e. pupil assessment) should inform and determine the aims and objectives in the next cycle, which would in turn lead to new selection of learning experiences and content, and so on.

Kerr (1968:17) went further in his model of a curriculum which stresses the inter-relationship between all the curriculum components. The model is depicted in figure 2.3 below:

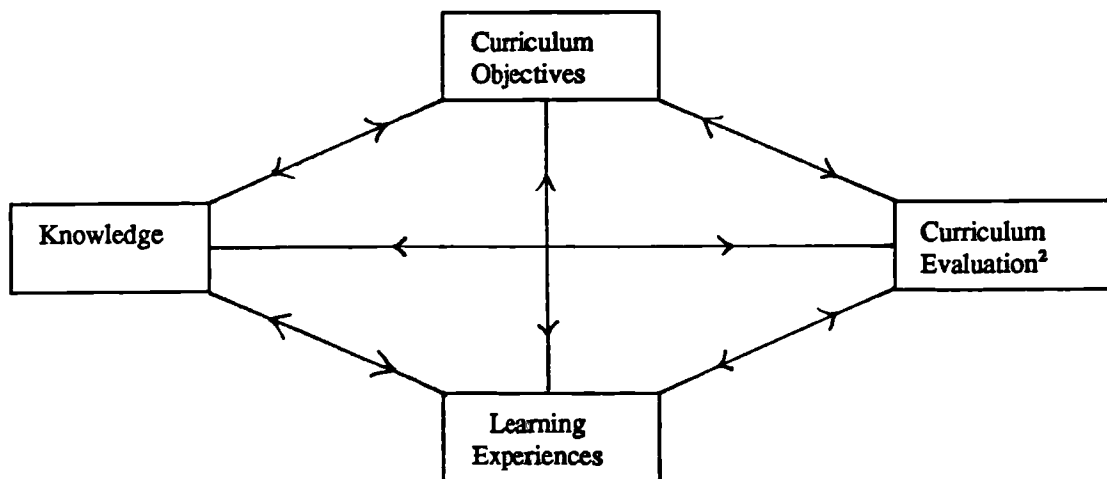


fig. 2.3: Kerr's Model

² Note that when Kerr used the term "curriculum evaluation", he referred to a component in the curriculum, the answer to the question "How are the results to be assessed?", rather than the evaluation of the curriculum as a whole.

Kerr's contribution is to pin-point the interactive nature of a curriculum. For Kerr, even Wheeler's cyclical conception of a curriculum is not dynamic enough. All the components of a curriculum, in Kerr's view, affect each other. The curriculum objectives, for examples, affect not only the knowledge chosen in the instruction, but also the learning experience to be organized for the students and the curriculum evaluation exercise. Evaluation, on the other hand, does not only inform the aims and objectives, but affects both the knowledge taught and the organization of the learning experience for students (including methods of teaching used).

Taylor (1967:167) suggested a three-dimensional model which stresses the various levels within the components. Taylor's model is represented like this:

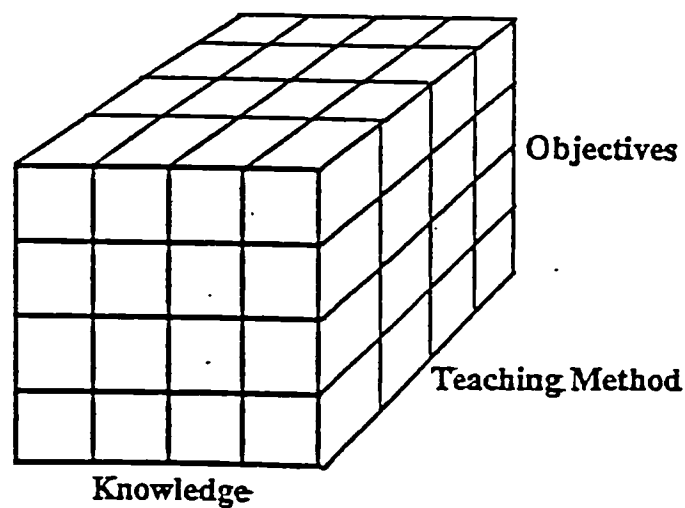


fig. 2.4: Taylor's Model

Taylor's contribution is his stress on the further breaking down of the curriculum components into levels or cells according to complexity. The Knowledge component of a curriculum, for example, is not just a simple list of content to be taught, but embraces "content" at different levels. According to Taylor, Knowledge "embraces knowledge arising from simple elements of experience to formal knowledge or knowledge of the disciplines ..." (Taylor, 1967:161). Similarly, Objectives range "from intellectual objectives such as the knowledge of some specific facts, ... to self-knowledge including knowing what one knows and how it is known" etc. So in studying a curriculum, it is important not only to pay attention to its components, but to the different levels of complexity within the components as well.

To add to this already abundant (or perhaps confused) repertoire of models, mention must be made of the contributions by the Second International Mathematics Study (SIMS) organized by the International Association for the Evaluation of Educational Achievement (IEA), not least because it is a study of the mathematics curriculum. The conceptual model used in the SIMS conceives a curriculum as being made up of three level (Travers, 1980): the intended, the implemented and the attained levels. The intended curriculum refers to intention at the educational system level. This information can be obtained from government officials and from officially published documents such as syllabuses, curriculum guides, circulars etc. The implemented curriculum refers to the curriculum as transacted in the classroom, and obviously, there is no reason to believe that this is necessarily identical to the intended curriculum. In fact, research and experience tell us

that there is usually a discrepancy between the intended and implemented curriculum³, and by conceptually differentiating between these two levels, the extent of the consistency between the two levels can be highlighted for study. The third level pertains to the curriculum as attained by the students, i.e. achievements of students, however achievements are defined. Again, experience tells us that there are usually discrepancies between student attainment and what the students are intended to learn and what they are actually taught in the classroom, and explorations on the discrepancies between the three levels would be very revealing to the educator who is interested to understand the "curriculum" of a place. The SIMS model can be represented like this (Travers, 1980:189):

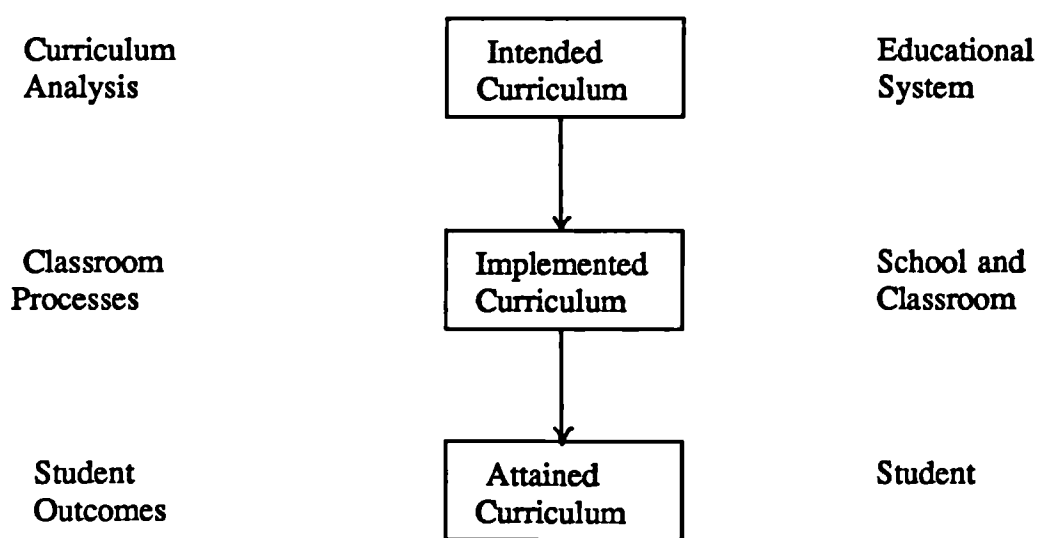


fig. 2.5: The SIMS Model

³ See for example Raynor's distinction between intended and transactional curriculum (Shipman and Raynor, 1972:78), Stenhouse's distinction between curriculum as intention and curriculum as reality (1975:2), and Eisner's distinction between intended and operational curriculum (1985:46).

The SIMS model, unlike the models designed for curriculum planning reviewed so far, is essentially a model developed for the purpose of research and evaluation, and it contributes in highlighting the importance of looking at three important but distinct levels in the curriculum, a fact that is often overlooked by some researchers. Very often, in conducting curriculum research, researchers use the same label to talk about things at quite different levels (for example, using the word content without making clear whether it is the intended content or the implemented content). The SIMS model has clarified the conception of this complex notion of a curriculum.

The brief survey above samples the efforts past educators have made in trying to arrive at a conceptual framework so as to capture the complexities of the notion of a curriculum. Each model reviewed has its own strengths, but simply combining them together does not yield an integrated model suitable for use in the present study. For example, Tyler's model (and its derivatives) and the SIMS model seem to have captured two important but different aspects of the curriculum, and suggest the direction towards a workable model. If the two models are taken as two different dimensions of a curriculum, a two way grid can be constructed accordingly as shown in figure 2.6 on the next page (See Steiner, 1980:7 and Stake, 1967:528).

But when the respective cells in figure 2.6 are actually examined, it is found that not all the cells are interpretable. What, for example, does the cell "implemented aims" mean? (Can aims be implemented?) Is "implemented teaching methods" any different from "attained methods"? So a simple laying out of a two-dimensional grid using the Tyler and SIMS models is not adequate for the purpose of the present study.

	Aims/Objectives	Content	Methods	Assessment
Intended Level				
Implemented Level				
Attained Level				

fig. 2.6: A Two-way Grid: Tyler's Model x SIMS Model

One way of arriving at a workable model of a curriculum is to improve on the two-way grid in figure 2.6 by deleting those cells that are not meaningful. The "improved" grid looks like this:

	Aims/Objectives	Content	Methods	Assessment
Intended Level				
Implemented Level				
Attained Level				

figure 2.7: An Improved Grid: Tyler's Model x SIMS Model

What are missing in this improved grid are input (teachers) and output (students) attitudinal factors of the curriculum. If these are integrated into the grid and the cells in the grid are rearranged, bearing in mind that each cell consists of different levels of complexity as suggested by Taylor, a curriculum model which hopefully embraces the strengths of the various models reviewed can be obtained. The model is represented schematically in figure 2.8 below:

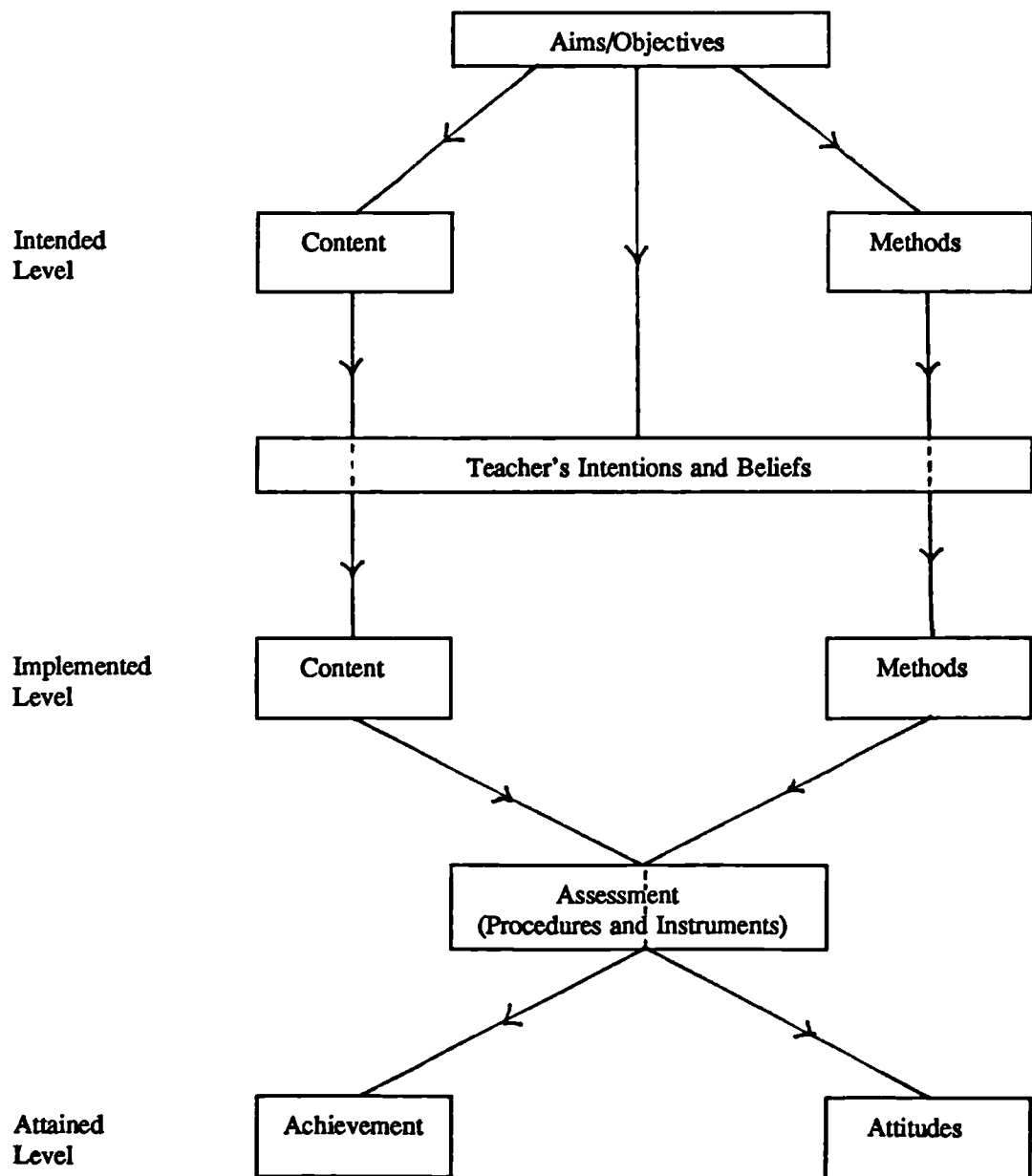


fig. 2.8: Model Used in the Present Study

In the model suggested in figure 2.8, there are three components at the (system) intended level. They are the aims and objectives as specified in the official documents, the intended content to be covered as set out in the official syllabus, and the officially intended methods to be used (if any). The first of the three components at this level determines (or has a logical priority over) the other two components, as indicated by the arrows from Aims/Objectives to Content and Methods respectively.

The implemented level consists of Content and Methods, and these two components are influenced by the corresponding components at the intended level mediated through the teacher's intentions and beliefs. One can argue that teachers' attitudes are part of the intended curriculum, but in this model, the intended curriculum refers only to the intentions at the system level. Notice that in the expanded model in SIMS (Travers, 1980:190), teacher beliefs and attitudes are placed at the implemented level. This practice is not followed in the present model because it is thought that what a teacher believes may not necessarily be implemented. In this model, teacher beliefs and intentions are perceived as mediation between the intended and implemented curricula, and so are placed between the intended and the implemented levels. By mediation, it is meant both when the teacher interprets the intention of the system and when the teacher holds a view that is different from the intention of the system. In either case, the influence of the intended curriculum on the implemented curriculum is only indirect. There could be no implementation without the implementer⁴.

⁴ except when the curriculum is in the extreme form of a teacher-proof programme-learning kind of materials, which is in practice non-existent in any large scale any where.

The attained curriculum comprises student achievement and attitudes, and these are affected by the teaching methods and content at the implemented curriculum level. The assessment component of the curriculum consists of testing procedures and instruments⁵, and is placed between the implemented and attained curricula. Studying the assessment procedures and test instruments is important both in understanding the intentions of the system and in interpreting test results of achievement and attitudes.

No originality is claimed for the model suggested. It is based on the curriculum models reviewed, but it is hoped that it embraces the strengths of the various curriculum models, and that it will prove to be a comprehensive and workable model for the present study.

⁵ as distinct from intended assessment outcomes, which are the same as intended content, and attained assessment results, which are the same as the attained achievement and attitudes

Past Studies

So far literature in the area of curriculum studies has been reviewed and a curriculum model developed for use in the present study. Next we turn to literature on past comparative studies in the mathematics curriculum and focus in particular on the methods used in these studies.

Lai (1968) wrote a report entitled "A Comparative Study of the Teaching of New Mathematics in Secondary Schools in Hong Kong and in the United Kingdom". The report was based on published documents as well as school visits in various parts of the U.K. It consisted of a short introduction to the education system in Hong Kong, followed by a brief description of the mathematics teaching practice in Hong Kong and U.K. respectively, the latter being illustrated by some anecdotal descriptions of the impressions gained from the school visits. Then came a brief survey of the New Mathematics movement, and the report ended by suggesting ways of implementing the new mathematics curriculum in Hong Kong.

The study can not be considered a comparative study in the strict sense of the term. It represents a typical study of the period (see for example Mak, 1967) when less-developed countries attempted to learn from the experiences of the more-developed countries. Relying mainly on impressionistic observations, the success of the developed country was usually highly exaggerated, and the pitfalls of the new programs overlooked. There was no attempt to collect data systematically, and the differences in contexts of the developed

and the developing countries were usually not well taken into account. The inevitable result was an attempt to transplant the whole curriculum from one place to another without due consideration of the complexities of the different contexts involved.

In a rigorous comparative study, care must be taken not to rely solely on impressionistic observations, and any "applications" of the comparative results must take into full account the likely differences in the cultural and societal variables of the different places.

Wang and Lu (1981) did a comparative study of the mathematics curriculum at the junior secondary level in various countries including Taiwan, Japan, U.S.A., France, U.K., West Germany and U.S.S.R. The purpose of the study was to "identify trends of curriculum development in various countries ... for reference in improving the curriculum and revising textbooks (in Taiwan)" (Wang and Lu, 1981:5). The study included a comparison of the aims of mathematics education, time allocation to mathematics instruction, content coverage and textbooks. The method used in the comparison was first a detailed reproduction of the relevant information and materials in each of the countries, followed by a listing of the similarities and differences. The report ended by giving suggestions on each of the areas studied for the Taiwan curriculum.

The materials on which the study was based were syllabuses and textbooks from the early seventies in various countries. Some materials (for example when citing the content coverage of the American curriculum) were even from the early sixties, so it is not surprising that the report was still dwelling upon issues relating to the modern mathematics movement. For example, the report started with a discussion on the

modernization of mathematics education, quoting from psychologists such as Bruner, Piaget and Gagne, and commenting on the new mathematics movement in the U.S. The conclusion reiterated the importance of modernizing the secondary curriculum, claiming that the three foundation stones in the modern trend of secondary school mathematics were "linear system, calculus and symbolic logic" (Wang and Lu,1981:227).

The limitations of this Taiwanese study are obvious. Although the report was published in 1981 when the modern mathematics movement had already receded in Europe and North America, the study failed to base its information on more recent sources. Much of the data quoted in the report were out-dated, and in some parts of the comparison, materials were chosen from different periods of time for different countries; hence the results are somewhat dubious. The comparison was also limited by studying published materials only, and so the implemented and attained levels of the curriculum were totally disregarded. Also, in the "analysis" part of the report, comparison was equated with a listing of similarities and differences. No attempts were made to interpret the similarities and differences within some theoretical framework. So it is very difficult for a reader of the report to draw meaning from the results. The report itself did not draw any conclusions from the results, and the "conclusion" of the report as mentioned above was a reiteration of the importance of modernization of the curriculum; it is difficult to find any relationship between the conclusion and the preceding findings. Nevertheless, the report did represent an effort on the part of Taiwan to improve its curriculum based on a study of the experiences of other countries, however limited that study was.

Keitel (1982) reviewed two comparative studies on mathematics education in U.S.A. and U.S.S.R.: *An Analysis of Mathematics Education in the Union of Soviet Socialist Republics* by Davis, Romberg, Rachlin and Kantowsky (1979) and an unpublished report by Wirszup (1981). According to Keitel, Wirszup "compares numbers" and the data "accumulate to give a sense of considerable qualitative and quantitative lags on all levels of secondary mathematics (and science) education in the U.S., with extreme differences being found among students not heading for college." (Keitel, 1982:111) The Davis *et al* report on the other hand intends to "provide information on Soviet mathematics education" and presents "individual views on mathematics education, research, and development in the Soviet Union." They "ultimately focus on the achievements of Soviet research in learning psychology and their implications for methodology of both research and educational practice." (Keitel, 1982:117)

Keitel's concern in the review was more on the different paradigms underlying the mathematics education and research in the two countries than on the actual curricula in the two places, which is the focus of the present study. The review however serves to underpin the importance of educational research guided by 'content' rather than excessively determined by methodology (see p.120). Hence, in comparing the two comparative studies, Keitel praised the "observational and discursive" approach used by Davis *et al*, an approach that was more consistent with the Soviet educational research paradigm. On the other hand, Keitel considered Wirszup's study "a good illustration of the technological approach to investigation" which treated mathematics education as if it has "natural-law-like objectivity" (Keitel, 1982:120-121).

For the present study, and indeed for any comparative studies which involve subjects from widely different cultures, Keitel's concern needs to be heeded. There is always the danger of employing a research methodology that is derived from one of the places under study and employing it indiscriminately to study the education in other places, regardless of wide differences in culture. What is needed in the present study is a sensitivity in choosing the research methodology so that the approach is appropriate for studying the situations in the three different places concerned.

The American mathematics educators Eckstein *et al* (1982) did a comparative review of the mathematics (and International Studies) curriculum at secondary school level in five countries including Canada, Japan, West Germany, the U.S.S.R. and the United States of America. The purpose of the study was to describe the curriculum content and provide relevant background information so as to "identify what students are taught at various levels and to highlight what appear to be noteworthy recent developments in the several nations observed." (Eckstein *et al*, 1982:1) The study limited itself to reporting on the intended curriculum alone, and also admitted that other dimensions such as the historical, the curriculum as a whole and other social factors were not considered.

The descriptions of the mathematics curriculum in the five countries were very brief. The average length in the report in describing the whole secondary school mathematics curriculum was only about one and a half pages for each country (the minimum was one page - for West Germany, and the maximum two and a half pages - for the U.S.). The report then went on to focus upon some themes for comparison. Drawing on the SIMS data and other sources, it first compared the time and grade when various mathematical

topics were introduced into the curriculum of different countries. The availability of higher mathematics topics in secondary school was also compared across countries and listed out in a table. The report then discussed some noteworthy programs and practices in Canada (curriculum revision in Geometry), West Germany (computers and Mathematics), Japan (quality of teacher-student transaction) and the U.S.S.R. ("New Math"), and concluded by drawing readers' attention to some issues in mathematics education in the U.S.A.

It is obvious that the "noteworthy" programs were chosen with the American agenda in mind, that they were noteworthy in the American perspective, and were not necessarily noteworthy in the respective places where the programs were found. Perhaps this bias in focus is inevitable, for all comparative educators would have their own agenda in mind when conducting the comparison. As long as this is made clear at the outset, it is perfectly justified to compare or to describe along some pre-determined lines. The peril to avoid is to present the different countries' situations according to the agenda of the researcher only, thus distorting the picture of the situations under study. Had the Eckstein study given a fuller description of the mathematics curriculum in the various places and pointed out the issues of concern in the various places (in contrast to the American concern) before presenting the selected programs, it might have proved to be a well-balanced comparative study of the mathematics curriculum of the five countries.

Chang (1984) did a comparative study of mathematics education in Taiwan and the United States. The comparison was based on the results of a survey questionnaire sent to teachers in the two places. The study encompassed a large number of variables on

schools systems, curricula (i.e. content, as the term was used in the report), teaching methods, and teacher training, with a disproportionate part of the report (about one fifth of the whole report) devoted to describing an in-service training program of primary school teachers in Taiwan.

The part of the report on comparing the curriculum (content) was a listing of the mathematical courses available in the U.S. and in Taiwan respectively, and there was no attempt to discuss or compare the lists except in noting (without explanation) that "the variety of mathematics courses from which students may choose at the high school level" in the U.S. had caused low achievement for American children (Chang, 1984:92). There is certainly more to a curriculum (even when restricted to the content component) than merely choices of mathematical courses, and the report seems to have ignored all other areas within curricular content.

As for the instructional methods, based on teachers' report through the questionnaires, Chang found that "teaching methods in mathematics are similar in the two countries. The majority of teachers use the chalk/lecture method to teach their classes" (Chang, 1984:41). No attempt was made to ascertain whether the actual practice was consistent with the teachers' report or not. Nor could readers of the report know whether the kind of chalk/lecture teaching in the U.S. was the same kind of chalk/lecture teaching in Taiwan or not. This criticism is probably not one that applies only to Chang's study but to any study that does not have a classroom observation component. But throughout Chang's report, there was no indication that this point was taken into account or acknowledged in the comparative study.

More interesting is a group of research projects known as the Mathematical Understanding of Taiwan students (MUT) that was recently carried out in Taiwan (quoted in Lin, 1988). These were not exactly comparative studies, but were replications and adaptations of the 'Concepts in Secondary Mathematics and Science' (C.S.M.S.) project in the U.K., and as such, the findings when compared with the C.S.M.S. results would yield valuable comparative data.

Lin (1988), in a discussion of some of the results of the MUT study, compared the performance of the Taiwanese students with their U.K. counterparts and found that they made very different kinds of mistakes. Lin tried to account for the differences in performance by the different characteristics of the languages used in the two places, and by the differences in emphasis concerning child-method versus taught method in the instructional processes of the two countries.

Lin's interpretation was only very tentative, but his contribution is his attempt to account for differences in the curricula using "extra-educational" factors. Past research tended to explain results within the educational realm (for example, explaining low achievement by the existence of too many choices of courses of study, as exemplified by the study by Chang cited above), as if the educational realm was itself a self-contained world. But international studies in education should remind us that the curriculum is in fact enmeshed in the greater societal and cultural context of a place, and it is impossible to give a comprehensive account of the differences and similarities in the curriculum without resorting to extra-educational factors.

I.E.A. Mathematics Studies

Probably the most comprehensive and large scale studies on comparison of mathematics curriculum were studies conducted by the International Association for the Evaluation of Educational Achievement (IEA). The first international mathematics study (FIMS) was completed in the sixties and involved twelve countries (Australia, Belgium, England, Federal Republic of Germany, Finland, France, Israel, Japan, Netherlands, Scotland, Sweden and the United States), while 20 countries or systems participated in the second study (Belgium - Flemish and French, Canada - British Columbia and Ontario, England and Wales, Finland, France, Hong Kong, Hungary, Israel, Japan, Luxembourg, Netherlands, New Zealand, Nigeria, Scotland, Swaziland, Sweden, Thailand and the United States of American), which was completed in the early eighties.

In a sense, the FIMS was more than a study of the mathematics curriculum. It was meant to be a comparative study of the outcomes in different education systems around the world using mathematics achievement as the indicator for output (McLean *et al* 1986: preface). As Husen put it, the main objective of the study was "to investigate the 'outcomes' of various school systems by relating as many as possible of the relevant input variables" (Husen, 1967a:30). Mathematics was chosen because of the general interest in the mathematics curriculum around the early sixties and because mathematics was considered a universal language, relying on symbols that were more or less "international" and hence easier for comparison.

The FIMS was a comprehensive study, collecting information on the educational systems, school structures, teacher attitudes and classroom practices, as well as student attitudes towards and achievement in mathematics. Achievement was measured by tests that were specially designed for the purpose through a thorough procedure which involved national centres comprising mathematicians and mathematics educators. Other information was basically collected through questionnaires, one for students, one for teachers, one for schools (school heads) and one for nations (completed by a representative in that nation).

The FIMS report was written by Husen (1967a, 1967b) and published in two volumes. For a study of this nature and scale, it is not surprising to find that many of the "hypotheses" posed at the outset of the study could neither be confirmed nor rejected. In the chapter on "Summary of Major Findings" in the second volume of the report, for example, the author admitted that "neither the 'productivity' of an educational system of a country, nor the effect of the instruction given, can be assessed from national means" (Husen, 1967b:288). The only consistent finding was the great variability in student achievement. The report listed the correlations of achievement with a lot of other variables, but without a more detailed understanding of these variables in the contexts of the countries involved, these correlation coefficients mean very little. For example, one of the findings was that "per-student expenditure or teacher training displays a sizable negative correlation with total mathematics score on the 13-year-old level", and the report attributed this finding to "the fact that Sweden and the United States have the lowest score and at the same time the highest per-student expenditure, while in Japan the opposite is the case" (Husen, 1967b:290). But the report did not go on and explore why this phenomenon occurred in the first place. The report also discussed problems related to

school organization, instruction and curriculum, and societal factors, and in general there were no important findings other than some rather obvious conclusions (e.g. "the mean level of mathematics score was lower among students from lower socio-economic status" (Husen, 1967b:303)). The proponents of the FIMS argued that these "obvious" findings had never been tested empirically in the international setting in a manner treated by the FIMS. This perhaps is the single important contribution of the FIMS.

The FIMS report drew the attention of the press in a number of countries, as well as criticisms from scholars within and without the discipline of mathematics education. Although the report emphasized that the study was not meant to be an "international contest" (Husen, 1967b:288), the public tended to look at it that way. Newspapers in countries like the U.S. filled their headlines with captions such as "U.S. Gets Low Marks in Math", and politicians capitalized on the propaganda and advocated changes in mathematics education. Within academic circles, strong criticisms were raised against the study. The *Journal for Research in Mathematics Education* devoted a whole issue to discuss the study (1971,2,1), but perhaps the strongest criticisms came from Freudenthal, at that time editor of the journal *Educational Studies in Mathematics*. Freudenthal (1975) complained that mathematics educators were not involved in the design of the study, with the result that curriculum variables were neglected. He argued that the tests used were not valid, that the meanings of many of the variables overlapped, and that some important variables such as examinations were not included.

Many of the harsh criticisms by Freudenthal were fair ones, but perhaps in a first-of-its-kind large scale international study like the FIMS, these mistakes were inevitable. In fact,

one contribution of the FIMS was that it enhanced international co-operation in educational research, and drew attention internationally to the importance and possibility of systematic comparative study of curricula. It is true that the study had a bias towards psychometric testing, but in that it reminded future researchers to place more emphasis on curricular issues and persuaded mathematics educators of their important role in this kind of study in the future, the study definitely marked an important page in the history of comparative studies of the mathematics curriculum.

The second international mathematics study (SIMS) by IEA was conducted in the early eighties. Like the first study, a large number of variables were studied in addition to mathematics achievement. But the organisers emphasized that, unlike the first study where mathematics was chosen "for convenience" (Travers, 1980), the second study was focused on the teaching and learning of mathematics in schools. Its purpose was "to compare and contrast, in an international context, the varieties of curricula, instructional practices, and student outcomes (both affective and cognitive) across the schools of twenty countries and educational systems." (Travers and Westbury, 1989:1)

Like the first study, a lot of information on the context of the curriculum in the participating systems was collected. For the curriculum itself, as pointed out in an earlier part of this chapter, the SIMS conceived it as being made up of three levels, the intended, the implemented and the attained levels. The first level was measured by survey questionnaires as well as an international grid. There were two dimensions to the grid. The first was a content topic dimension which included areas in Arithmetic, Algebra, Geometry, Probability and Statistics, and Measurement for population A (children of age

13), and Sets, Relations and Functions, Algebra, Geometry, Analysis, Probability and Statistics, and Finite Mathematics for population B (the last grade before entering university). The second was a "behavioural categories" dimension which included Computation, Comprehension, Application and Analysis. Educators in different countries were asked to rate each cell in the grid as very important, important, some importance and not important. From the results of these completed grids, measures of how well the study fitted the curriculum in particular countries as well as cross-country comparisons could be made.

The implemented curriculum was measured by, among other instruments, an Opportunity to Learn (OTL) index which asked teachers to rate each item in the tests given to their students according to whether that topic had been taught to the students or not. From the OTL data, the consistency between the intended and implemented curricula of a country could be judged, and the students' "opportunity to learn" various topics in different countries could be compared.

Lastly, like the first study, student achievement was measured by a battery of tests, and attitudes measured by questionnaires, with some of the items identical to those in the first study for comparison purposes.

The major findings of the second study were that, for the education systems, "there is a fair degree of structural similarity between the various target populations even though there are significant differences in the organization of curricula and courses and in the grade levels being investigated" (Travers and Westbury, 1989:78). For the intended

curriculum, there was a "common core" of mathematical content for all countries, but there was great variety in Geometry content across the systems (p.110). For the implemented curriculum, there was both between- and within-system diversity in content (p.165), but teachers' general approaches to the teaching of mathematics were rather similar, namely largely "chalk-and-talk" and relying heavily on a prescribed textbook (Robitaille and Garden, 1989:235). Furthermore, there were substantial overall discrepancies between the intended content coverage and implemented coverage (OTL), so that it was "difficult to make confident claims about the validity of either or both of these indices of coverage" (Travers and Westbury, 1989:116). For the attained curriculum, "performance on items involving ... straightforward applications of basic concepts was generally good; however, performance fell off sharply on items calling for the use of higher order thinking skills" (Robitaille and Garden, 1989:238).

Eight of the participating countries in the second study also undertook a "longitudinal" study which included a pre-test to students and a detailed classroom process questionnaire to teachers dealing with how they handled subject matter during the year (Travers and Westbury, 1989:4). Unfortunately the report on this part of the study was not available at the point of writing this review.

Compared with the first study, more attention has been put on the curriculum in the second study, especially the emphasis on the implemented curriculum. However, despite this emphasis, the study relied on teachers' report (through OTL and other questionnaires) on the way the curricula was transacted in the classroom, and there was no attempt to validate the reported practice through classroom observation. The limitations of teacher

self-reports are obvious (this will be elaborated in chapter three), and for the SIMS the problem was even more serious because the findings pointed to doubts concerning the validity of these instruments.

Perhaps for an international study of the scale such as the IEA studies, these drawbacks are inevitable. For the present study, similar to SIMS, the curriculum is the major focus. Furthermore, because of the smaller scale of the study, it is possible to collect data on the implemented curriculum through classroom observation in the places under study.

England participated in both the FIMS and the SIMS, Hong Kong only participated in the SIMS, while China was involved in neither. The England and Wales SIMS report (Cresswell and Gubb) and the Hong Kong report (Brimer and Griffin) were published in 1987 and 1985 respectively. Since these were essentially national reports, the findings in the two places could not be compared directly. The Hong Kong report did relate the Hong Kong results to the international norms, but unfortunately, the England report was only concerned with the analysis of the English curriculum, and no attempt was made to compare the results with other countries or the international norms. So no comparison between the English and Hong Kong curricula could be made based on these reports.

It is interesting to note that although the SIMS international reports in general do not discuss the curriculum in individual countries or systems, there is a paragraph in the last chapter of the second report (Robitaille and Garden, 1989:236) under the section Major Findings which comments specifically on the situation in Hong Kong:

Some of the largest class sizes were found in systems where performance levels of students were among the highest in the study, and particularly in Hong Kong and Japan. Students in Hong Kong were not only in some of the largest classes found in the study, but they were also among the youngest participants and they were among the most likely to have teachers who were less than fully qualified to teach mathematics. The performance levels attained by Hong Kong students given these kinds of handicaps were outstanding, and they underscore the important role played by motivation and parental or societal encouragement.

This study will perhaps help in illuminating the reasons for the superior performance of the Hong Kong students under the relatively disadvantaged circumstances.

The University of Michigan Studies

More recently, there has been a series of studies based at the Centre for Human Growth and Development of the University of Michigan comparing academic achievement of children in Japan, Taiwan and the United States. The first study was done in 1979-80, on grade one and five pupils in the cities of Sendai, Japan; Taipei, Taiwan and the Minneapolis metropolitan area, US. Data were collected through interviews with parents, classroom observations and cognitive tests. For mathematics, the tests included computation for year one and year five students, and geometry for year five only. The classroom observations employed a systematic time-sampling method "in which observers coded the presence or absence of predetermined categories of (pupil and teacher)

behaviour" (Stigler, Lee and Stevenson, 1987:1274) for collecting data. The target (either teacher or student) was observed for 10 seconds, and then the observer spent the next 10 seconds for coding the presence or absence of behaviour from a checklist of categories. There were 30 categories in the student and 19 in the teacher coding systems.

The results of the achievement tests show that pupils in the two Asian cities outperformed their American counterparts. As far as classroom practice is concerned, the results show that Japanese and Taiwanese children spent "significantly more time learning mathematics" and "engaged in academic activities than did U.S. children" (Stigler and Perry, 1988a:207), who spent more time "engaged in inappropriate, off-task activities" (p.212). Classrooms in Japan and Taiwan were also found "highly organized and orderly; those in the United States more disorganized and disorderly" (p.209), and because of the differences in organization, "U.S. students experience being taught by the teacher a much smaller percentage of time than do the Asian students, even though U.S. classes contain roughly half the number of students" (p.211). These may have resulted from the fact that Japanese and Taiwanese teachers spent most of their time working with the whole class, "imparting information about mathematics to their students" (Stigler, Lee and Stevenson, 1987:1281) rather than letting students work in small groups or individually, which was more prevalent in the U.S.

The second study was designed "specifically at understanding the cross-cultural differences in mathematics learning". Data were collected in 1985-86, in Sendai, Japan; Taipei, Taiwan and the Chicago metropolitan area. The pupils, their mothers, teachers and school heads were interviewed; mathematics lessons were observed and a series of tests

including word problems, operations, visualization, graphing, mental calculation, number concepts, estimation, and mental image transformation were administered. For the classroom observations, in addition to the "objective" coding system, narrative descriptions were also recorded by a second observer.

Results of the second study are similar to those found in the first. In addition, from the preliminary analyses of the narrative records, it was found that "both Chinese and Japanese classrooms provide more opportunities than U.S. classrooms for students to construct a coherent representation of the sequence of events that make up a typical mathematics class and to understand the goals of the activities in which they are engaged" (Stigler and Perry, 1988a:213). Also, Taiwanese classrooms were found to emphasize performance, Japanese classrooms were found to emphasize reflection and verbalization, while the U.S. classrooms "appear confused in this regard, and accomplish neither goal well".

The contribution of the Michigan studies is its stress on how mathematics is taught in classrooms in different cultures. It is the only study reviewed so far which has a classroom observation component in its data collection, a methodology which is certainly more valid than relying on teacher reporting on their own classroom practice.

The obvious drawback of the first study is in its classroom observation methodology. The limitations of systematic time-sampling methods will be discussed in chapter three of this thesis, suffice to say here is that no mechanistic coding of occurrence of behaviour based on pre-determined categories is able to capture the complexities of the implemented

curriculum in the classroom, especially when it involves classrooms in different cultures. The researchers themselves admitted that "data obtained from this method do not reveal some of the more subtle, but important differences in the ways in which mathematics classes in the three countries are conducted" (Stigler, Lee and Stevenson, 1987:1284).

The second study, with the inclusion of narrative descriptions in its classroom observations, is methodologically an improvement on the first, and as a result, more substantive differences between the classrooms in the three places seem to have been identified. Unfortunately, the full report on the second study was not available when the classroom observation data of the present study were analyzed, and so the analysis could not draw on this second Michigan study more.

There are two limitations which apply for both studies. Firstly, the studies involved countries where three different languages were used, yet most of the researchers were not trilingual. So the studies had to rely on different researchers collecting data in different countries, and on employing bilingual and trilingual coders to summarize the classroom observation data "in English for the other members of the research group" (Stigler and Perry, 1988a:213). This problem is not unique to the Michigan studies, but inevitable consequences of this limitation are problems of validity of translation and observer-observer reliability in data collection. For the present study, the researcher is conversant with the languages of instruction used in the three places under study, and so the problems mentioned above are minimized.

Secondly, the studies limited themselves to students in the first and fifth grades at the primary school level only. It is acknowledged that early years of formal schooling are important years in a child's development and education, but as argued in chapter one, the junior secondary years are also an extremely important stage for children, especially where their mathematics learning is concerned. The present study aims at illuminating the mathematics education at this important stage of junior secondary school in the three places concerned, and hopes to build on the results of the Michigan studies.

Summary

From the wealth of comparative study results just quoted, many lessons concerning comparative studies of the curriculum could be learned. First, a comparative study of the mathematics curriculum must be a 'study'. It has to be based on rigorous (but not necessarily mechanistic) design and planning, and data collection must be systematic and relevant. The information gathered must be based on up-to-date materials, and the data from the countries being compared must come from roughly the same time period.

Secondly, comparison should not be just a simple listing of cases, but should always involve analysis and interpretation along certain themes. The researcher is justified in choosing a theme of his or her own interest, but due consideration must be given to both the concerns of the educators of the places under study and any patterns that evolve from the data.

Thirdly, there is a need to concentrate on curriculum matters, although information about the education systems is also essential. It would be misleading if outcomes are solely or mainly related to extra-curricular variables without due consideration of the teaching and learning that takes place in the mathematics classroom. In particular, it is important to observe what actually happens in the classroom, for "to critique or appraise the operational (i.e. implemented) curriculum requires one to be in a position to observe what classroom activities actually unfold." (Eisner, 1985:47) In only one of the studies reviewed was there a component of classroom observation, and obviously, no matter how well questionnaires to teachers and students were set, the data collected could not substitute for the information gathered through classroom observation. So if the variables at the implemented level of the curriculum are really taken seriously, there must be a component of classroom observation in comparative studies.

Lastly, in collecting data and in interpreting results of the comparative study, the context or culture of the countries under study must be taken into account. Employing methodologies that are insensitive to cultural differences may yield biased results that distort the real picture of the curricula in those countries, and interpretation without regarding the cultural differences of countries being compared is inadequate or may even be misleading.

It is hoped that the present study would contribute to the literature an example of a comprehensive and balanced study, albeit a small scale one, and fill in some gaps of knowledge that now exist in the comparison of the mathematics curriculum in the three places concerned.

Chapter 3: Methodology

In chapter two, a curriculum model was developed based on a review of the relevant literature. The model defines what is meant by a curriculum in this study and determines the scope of what is being compared. In this chapter, exactly what kinds of data were collected and how they were collected will be specified. Then the way the instruments for collecting data were developed and piloted will be discussed, and the manner the main study was carried out will be described.

Data Collection

The first two groups of data collected are data on the contexts of the curricula and the system intentions in the three places. The former includes the cultural setting, the education system, and the history of development of the mathematics curriculum in each of the three places. System intentions in our model include three elements: aims, methods of teaching, and content.

Information on the cultural setting was obtained through a review of the relevant literature. Data on education system, history of development of curriculum and system intentions were gathered through documents, supplemented by interviews with the people concerned.

To describe the contexts of the curricula and the intentions of the curricula, two types of documents were studied: documents on the history of development of the mathematics curricula in the three places, and documents concerning the present curricula. These include: examination and teaching syllabuses, public examination papers, curriculum guides, circulars from the education authorities, government reports and research reports, journal articles, textbooks and other books written on the subject.

Supplementing the documentary data by interviews was especially important for the cases of China and Hong Kong where there were relatively few documents or journal articles on the history of development of the mathematics curricula. Literature concerning the present curricula was also limited to government publications. So in order to reconstruct the history of development of the mathematics curricula in these two places and to collect the views of educators on the present curricula, interviews with the relevant personnel were essential. For England, there were more relevant documents available, and so the data collection depended less on interviews. A list of the people interviewed in this study is given in Appendix A.

The next kind of data gathered are those on the intentions of teachers. Teachers, of course, have many different intentions that affect their teaching of mathematics: the reasons why they take up teaching as their profession, their views of education, their views and knowledge of mathematics, their knowledge of and attitudes towards their students, and their attitudes towards the environment that they are working in etc. Among these "intentions", most pertinent are their views of mathematics and of mathematics

teaching and learning. Indeed these two variables were often used in past research that tried to relate teacher intentions to teacher performances and student achievements. The SIMS (Robitaille and Garden, 1989:203-7), for example, measured among other attitudes, teachers' views on mathematics as being an interesting or easy subject, and also teachers' attitudes on "mathematics as a process" (this will be further elaborated below). Lerman (1986), in studying the relationship between teachers' conceptions of mathematics as being an "open" or "closed" subject and their actual performances in the mathematics classroom, also measured teachers' views on mathematics; while the second Michigan Study (see chapter two of this thesis) interviewed teachers about their beliefs on learning mathematics (Stigler and Perry, 1988a:205).

Teachers' intentions can be gathered either through interviews or questionnaires. Each of the two methods has its own distinctive advantages as well as limitations. Interviews can be used to probe the intentions of teachers in greater depth, and are more flexible if used tactfully by the interviewer. The limitations of interviews are that they are time-consuming; and if more than one interviewer is involved, inter-interviewer reliability is difficult to establish and the subjective element in the interview is difficult to detect.

Questionnaires can be administered on a larger scale involving less manpower, and once the questionnaire is set, the scoring and analysis of results are relatively more objective¹. The limitations of using a questionnaire lie in its inflexibility and in the difficulty in

¹ There cannot be total or absolute objectivity. Even in a questionnaire, the subjective element is present at the designing of the questionnaire items, their interpretation by the respondent, and in the interpretation of the results.

knowing the conditions under which the questionnaire is answered, and hence unacknowledged ignorance and unconscious biases are difficult to detect.

The original intention of the present study was to use both methods so that the results could complement each other. However, during the main study, various difficulties were encountered when the teachers were interviewed (see the section under Main Study in this chapter). Hence data on teacher intentions had to rely more on the questionnaire; the informal interviews with teachers conducted were only used to supplement the information gathered through the questionnaire. How the questionnaire was designed and administered will be discussed in later sections of this chapter.

The last group of data collected are those under the implemented curriculum in the curriculum model. As pointed out in the previous chapter, one characteristic of the methodology used in the present study is a component of classroom observation. Many past studies on curriculum comparison ignored the implemented curriculum altogether, and other studies which included a comparison at the implemented level usually relied on teachers' self-report of the teaching process. The problems of a self-reporting methodology are obvious. Firstly, the teacher may not be honest in the self-report. Even when the researcher promises confidentiality, the teacher may still fear that the report would be disclosed to people to whom he does not want it disclosed (for example, parents or the head teacher) and so may refrain from responding honestly. On the other hand, the teacher may also be sub-consciously self-defensive, not wanting to admit to himself and others that he is not performing as well as he has wanted to. These problems are

universal but are especially acute among Chinese people, who are in general very concerned about "losing face" in front of outsiders. (See Bond and Hwang, 1986:243ff)

Even when the teacher is honest in reporting his practice in the classroom, he may be sincerely mistaken about his own performance. Many teachers, in reporting on their classroom practice "honestly", have in fact subconsciously reported what they wanted to achieve rather than what they actually did.

Classroom observation, to a great extent, solves the above problems. Of course, the external observer cannot be totally objective either, but at least, he has the advantage of having no need to teach and evaluate the teaching at the same time, and in general has no vested interest in defending the teacher and so can be more objective compared to the self-reporting teacher. The disadvantage of classroom observation is that the presence of the observer may affect the teaching observed and thus distort the data collected. But on balance, it is considered a more valid method than self-reporting by teachers. For the present study, how the classroom observations were done, exactly what data were collected during classroom observations and the instruments for recording the lessons will be described later in this chapter.

From the classroom observations in this study, it was found that teachers in all three places relied heavily on the textbooks in their teaching (see chapter seven). So analyses were performed on selected topics in the most commonly used textbooks in the three places to throw light on the mathematics contents in the implemented curricula (see chapter eight).

To summarize, the various methods used in collecting data for the various components of the curriculum are shown in figure 3.1 below:

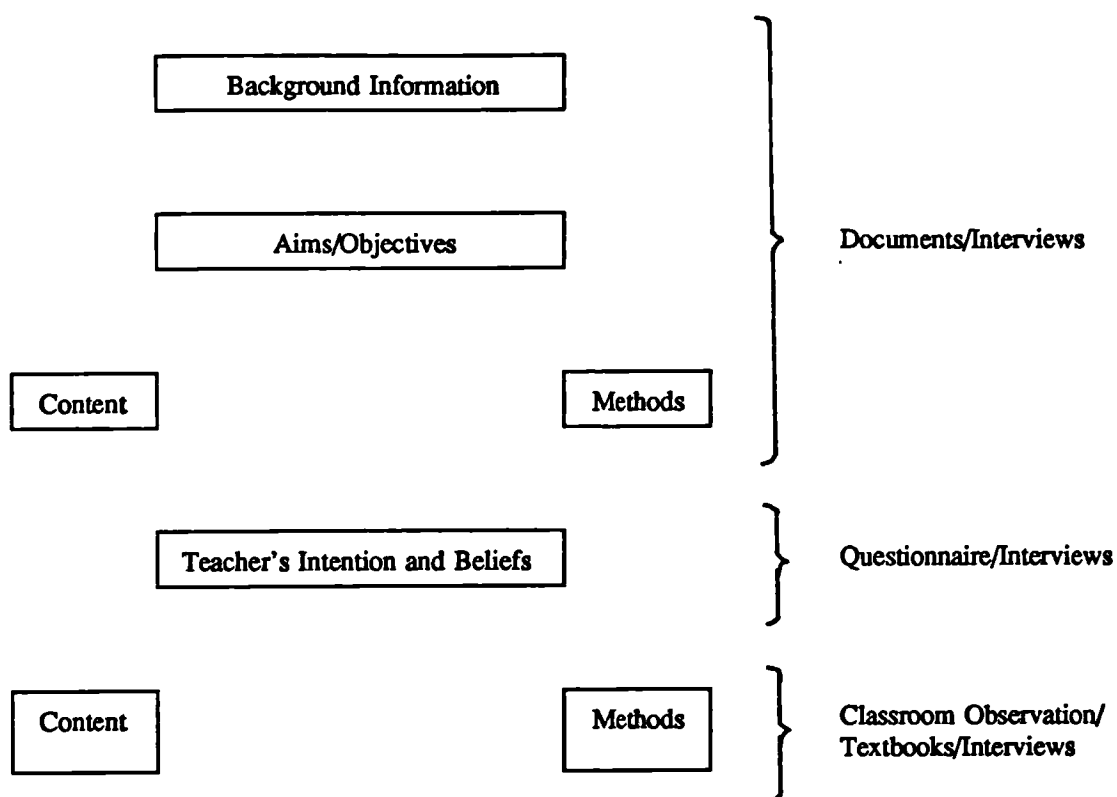


fig. 3.1: Methods of Data Collection in this Study

The Preliminary Visits

Before the development of the questionnaire and classroom observation schedule, schools in the three places were visited, mathematics lessons were observed, and teachers were interviewed informally to get a feel of what the mathematics teaching was like. These visits, observations and interviews were also important in helping to formulate the focus of the research.

The preliminary visits in London were done between November and December of 1987. Altogether 22 lessons in 4 different schools were observed. The researcher also had the chance of talking to some of the teachers, and brief field notes of these visits were taken.

There was a variety of teaching practices in the four London schools visited, with the private independent school teaching in a more traditional manner than the maintained comprehensive schools. In one comprehensive school, there were even great within-school differences in teaching styles between teachers. The lessons usually took place in a room designated for mathematics teaching only, with the students sitting in groups of 3 or 4 rather than sitting in rows facing the teacher. The students usually had many opportunities to work on their own, and in some cases, an individualized programme was followed and students in the same class studied completely different things. The lessons that were observed usually started later than scheduled, and during the lesson, there were very often incidents when some students were not paying attention to the teacher or to what they were supposed to be doing; they might be doing things unrelated to

mathematics or simply mucking about. The researcher's general impression of the mathematics teaching in London was that it was done in a relatively relaxed and free atmosphere.

The preliminary visits in Hong Kong were done in April of 1988. Altogether 7 lessons in 3 different schools were observed. Hong Kong is the place where the researcher was born and educated. The researcher also taught mathematics in Hong Kong secondary schools for a number of years and is now working as a teacher trainer there, so he is very familiar with the Hong Kong mathematics classroom. Although the preliminary visits were done with the present study in mind and the researcher tried to be as "objective" as possible during the observation, nothing new struck the researcher.

Perhaps the description in the SIMS Hong Kong report best summarizes the classroom situation in Hong Kong. The relevant paragraph is quoted below (Brimer and Griffin, 1985:23):

Perhaps stemming from the complexities of the system, the schools in Hong Kong have a reputation for reliance on teacher dominated instructional strategies. Pressures arise from the external examinations, the complexities of languages, expansion of the school system at the secondary level and consequent shortages of facilities including trained and qualified teaching staff. There has developed a tendency for classes to be taught by lecture-style delivery, with little student participation, apart from note taking and completing assigned work. ... The intense pressure of examinations, the expectations of parents, pupils and colleagues appear to encourage teachers to impart knowledge and instill in students a need to learn for a predominantly recall mode of performance.

Preliminary visits in Beijing were done in June 1988. Altogether, 8 lessons in 4 different schools in a certain district of Beijing were observed, 2 lessons in each school. After the classroom observation in each school, there were opportunities for the researcher to talk to the teachers, and in one school, a meeting was arranged with some students.

The mathematics lessons in China took place in the morning, usually one lesson per day every day from Monday to Saturday. The duration of the lessons was typically 45 minutes. As a common practice, the teacher would arrive at the classroom one or two minutes before the lesson started, and when the bell rang the lesson began immediately. The lesson would last right till the 45th minute.

The lessons were all predominantly teacher-lecturing, with some questioning and some time for class exercise. One salient feature of the teaching was the strict adherence to the textbook. The researcher was very surprised to find that when lessons at the same grade level but in different schools were observed on consecutive days, it seemed that the lesson in the second school followed the lesson in the first one exactly. For example, when the teacher started the lesson by summarizing the content taught in the last lesson, it seemed to the researcher that the teacher was summarizing the teaching on the previous day in the other school! When the teachers were interviewed, they admitted that they were following the textbook and the Teacher's Guide (see chapter five) very closely. They could even talk about a certain lesson, say "the lesson on addition of fractions with different denominators", as if all would understand exactly what that lesson should cover, what examples were used and what class exercise were given to students etc. A further reason for this uniformity of teaching in schools in this district of Beijing was that the education

authority of the district was trying to "upkeep the standards" of the schools, and one means of doing that was by setting standard tests for all schools in the district at regular intervals. In order for school children to be able to take these tests at the same time they had to proceed at a uniform pace. The researcher learned that schools were told to teach up to a certain line in a certain page of the textbook (of course they used the same textbook in all the schools) by a certain day! How this could be done for students with different background was difficult to understand (and in fact the 4 schools visited by the researcher had very different intake of students in terms of the academic achievement in their primary school), and whether the students could all achieve up to the expected level needed to be probed into further. But the teaching in the 4 schools observed was very similar, and it seemed in general that the students were able to follow the lessons well.

As a result of this uniformity of teaching in Beijing, it was not practicable for the researcher to study the teaching of one particular topic (this was the original plan of the research) if the classroom observations were to be done by one single researcher, for all (or at least most of) the different schools in Beijing would be teaching the same topic on the same day.

During this preliminary stage of the study, teachers from the three places were interviewed informally on their views on the intended and implemented curriculum. It was hoped that these informal interviews would help the researcher to arrive at a more structured schedule for future interviews. The teachers in London and Hong Kong were in general very open in expressing their views on the intended curriculum and the difficulties they encountered in the implementation of the curriculum. Some difficulties were however encountered in

interviewing the teachers in Beijing. Beijing teachers were in general more reserved and less willing to express their personal views on the curriculum. This was especially so when the interviews were done with a group of teachers instead of to individuals. In a group setting, a "spokesman" usually evolved (usually the mathematics department head, and in one case the head teacher, who happened to be a former mathematics teacher), and dominated the answers to the questions raised by the researcher. Even when questions were addressed to individual teachers in this setting, the teachers dared not express views contrary to those expressed by the spokesman, or simply referred to the spokesman for an answer. This defeated the very purpose of the interview, which was aimed at getting the views of the teachers (rather than the "official" view) on the curriculum. A way of getting over the reservation of the Beijing teachers was found when the questionnaire was piloted (see below). To avoid the problem of conforming to the "official" view, the researcher would request that interviews during the main study be conducted at an individual basis rather than in a group setting, but as can be seen later, this proved not to be possible.

Summary of the Preliminary Visits

It is obvious from the preliminary visits that what was happening in the mathematics classrooms of the three places differed tremendously. One obvious focus to look at in the main study is the kind of activities (and their frequencies and durations) that were happening in these classrooms. Another focus worth pursuing concerns the continuum between conformity and individuality. It seems that the Chinese teachers conformed more

to the official curriculum and that they demanded students' conformity to their teaching more (see chapter four), while there was more variety in teaching and learning in London. The Chinese teachers also seemed to stress rigorous mathematical language much more than their counterparts in the other two places, and this may be another focus of attention. Also, when talking to teachers from Hong Kong and Beijing, they admitted that their teaching was very much dominated by public examinations and uniform tests (even for the junior secondary level), much more so than teachers in London. This is yet another focus that can be looked into.

To sum up, the preliminary visits helped the researcher greatly in identifying the foci for comparison, and also informed the conduct of the pilot study and the main study.

Development of the Instruments for Data Collection

As pointed out above, data for the present study were collected through a questionnaire, classroom observations and interviews. On the basis of the literature review and the preliminary visits, three instruments were designed: a questionnaire for measuring teacher attitudes, a schedule for recording classroom teaching, and an interview schedule for interviewing teachers. The manner in which these instruments were developed is described below:

Questionnaire

In this study, a questionnaire was to be used to assess teachers' beliefs and intentions: their views of mathematics and of the mathematics teaching and learning process. To develop such a questionnaire rigorously would be beyond the scope of the present study, and what the researcher did was to adapt existing questionnaires for use in the study.

Questionnaires measuring people's views on mathematics abound (see for example Aiken, 1969; Bracht, 1972; Jones, Henderson and Cooney, 1986). Mention has already been made of the SIMS instruments and Lerman's questionnaire. Moreira (1992) also designed a questionnaire which measures, among other things, teachers' attitudes towards mathematics. The Moreira and Lerman questionnaires have their distinctive emphases respectively, which are not exactly the focus of the present study². Moreover, they both have the disadvantage of being too lengthy: the former has 21 items while the latter has more than 40. The SIMS "Mathematics as a Process" scale, on the other hand, is relatively short (only 15 items) and has been widely used in a cross-cultural setting, and it seems to be measuring a very important aspect of teachers' view on mathematics. So the "Mathematics as a Process" scale was adopted to be part of the questionnaire for the present study. The "Mathematics as a Process" scale consists of 15 statements about mathematics, and respondents are asked to indicate their agreement or disagreement with

² Moreira's concern was on the effects of a Logo-based mathematics in-service course on mathematics teaching, while Lerman was interested in teachers' conception of mathematics as being an "open" or "closed subject."

each of the statements. The scale is supposed to assess people's "perceptions of the nature of mathematics as a discipline". It measures "the extent to which (people) view the discipline as rule-oriented or heuristic, as fixed or changing, and as a good field for creative endeavour or not." (Robitaille and Garden, 1989:199)

There are relatively fewer existing questionnaires that measure views on the teaching and learning of mathematics. Moreira's questionnaire includes a section of 19 items that measure teachers' attitudes towards the teaching of mathematics, but the questionnaire was designed for primary school teachers and the items do not seem to fit the purpose of the present study. It seemed that there was no suitable questionnaire available in this area, and so the researcher decided to design items on his own. Based on a literature review (see chapter four) and the preliminary visits, the following themes were hypothesized to reflect contrasts between the views of teachers from the three places and were thus chosen for study:

1. the perception of mathematics as being a rigid subject
2. the stress on memorization
3. the stress on conformity (to rules and to teachers' instructions), and this is related to the encouragement or otherwise of differential treatment of students
4. the stress on practice
5. the place of mathematics in the overall curriculum
6. the aims of mathematics education
7. the attribution of success and failure in mathematics

8. the determinants of content of teaching, and
9. the perception of mathematics as a male domain

Themes 1, 2, and to a certain extent 3 were covered by the "Mathematics as a Process" scale. The remaining themes were measured by items designed by the researcher. After a few trials of drafts of the questionnaire in Hong Kong, a final draft was prepared for piloting. The final draft of the version for London teachers is given in Appendix B.

The final draft consisted of two sections. Section A asked for the teacher's background information including gender, age, qualifications, training, teaching experience, and workload. Section B and item 8 of Section A attempted to measure teachers' views on mathematics and mathematics education.

Items 9 to 23 of Section B were taken from the SIMS "Mathematics as a Process" scale. In addition, teachers were asked whether they agreed with the following statements or not: "Students in the same class but of different abilities should learn different things in mathematics" (Section B, item 7); and "Students of roughly the same standard should be grouped together in sets for instruction" (Section B, item 8).

On theme 4, teachers were asked the average amount of time they expected their students to spend on mathematics homework (Section A, item 8). On theme 5, teachers were asked to rank order a given list of common school subjects, and the rank for mathematics would be noted (Section B item 1). For theme 6, a review of the literature showed that in general aims of mathematics education fall within five categories (see for example:

Butler *et al*, 1970; Cockcroft, 1982:1-4; Cornelius, 1982:38-9): application to everyday life, tools for other subjects, training of the mind, appreciation of mathematics as part of our cultural heritage and for its intrinsic worth or beauty, and the ability to communicate logically and concisely. Teachers were asked to rank order the five (Section B item 2).

For theme 7, teachers were asked what the most important factors affecting students' success or failure in mathematics were. The factors they were given to choose from were derived from Attribution Theory (Weiner, 1971, 1979) and included: the ability of the students (uncontrollable, internal and stable), the effort paid by the student (controllable, internal), the teacher's teaching (controllable, external, stable), luck (uncontrollable, external, unstable), and the support from the student's family (uncontrollable, external, stable) (Section B, item 3). They were also asked whether they agreed with the statement "All children irrespective of their abilities can learn mathematics well" or not (Section B, item 6).

On theme 8, teachers were asked directly what should have the greatest influence on the mathematics taught in the classroom. Again they were given five options to choose from: the syllabus, the textbook(s), whether the content is interesting and meaningful for students, whether the content helps to train students' minds or not, and whether the content is useful for students in their future or not (Section B, item 4). And on the last theme, teachers were asked whether they agreed with the statement "Boys are better than girls in mathematics" or not (Section B, item 5).

For items 5 to 23 in Section B of the questionnaire, a five point Likert scale (Strongly Agree, Agree, Neutral, Disagree, Strongly Disagree) was used, following the practice of the SIMS instrument. The alternative of using a four point scale so that respondents were forced to take sides had been considered. The advantage of the latter was to avoid respondents' tendency to choose a neutral position. But for some of the items in this questionnaire, it was envisaged that some teachers would find it difficult to take sides because they might not know much about the subject (e.g. items 12 and 20 of Section B). This was confirmed during the interview with teachers after they had completed the questionnaires in the pilot study. Moreover, following the practice of the SIMS would have the additional advantage of the possibility of comparing the results of this study with those of SIMS. So it was decided that a five-point Likert scale be used for these items.

Items 1 to 4 in Section B required teachers to rank order statements according to their perception of the importance of the different options. There had been a temptation to convert these items to conform to a five-point Likert scale format as well. In fact, the Moreira questionnaire did use a five-point Likert scale, and in measuring teachers' view on the aims of mathematics education for example, five aims were stated as five separate items and teachers were asked to rate each on a five-point scale. The difficulty with this practice was the problems envisaged at the stage when the results were analyzed. For example, a teacher might feel that all five aims were important, and so might rate all five as "Strongly Agree" or "Agree". But for a comparative study, what is more important is whether there are differences between teachers from different countries on the RELATIVE importance of the aims, and so it is more appropriate for teachers to be asked to rank order the options. In so doing, it was also hoped that the teacher would put in more

thoughts when completing the questionnaire. Therefore in the final draft of the questionnaire, teachers were asked to rank order the first 4 items in Section B but tick on a Likert scale for the rest of the items. Thus the item type of the questions attempted to meet the purpose of the questions.

The inclusion of items that required teachers to rank order would of course have the disadvantage of requiring of the respondent more time to complete the questionnaire. But since this was a relatively short questionnaire, and there were only four items of this nature, it was thought that the questionnaire would be acceptable to teachers.

The questionnaire was translated into Chinese for use in Beijing. The Chinese version of the "Mathematics as a Process" scale had already existed since Hong Kong participated in the SIMS and Chinese versions of the instruments were used for Hong Kong students. The other parts of the questionnaire were translated by the researcher himself and checked by a professional translator. As the researcher was familiar with both languages, it was thought that checking through back translation was not necessary. A bilingual Section B version was used for Hong Kong teachers in the pilot study, to make sure that the teachers really understood the statements, and for the purpose of asking Hong Kong teachers to check the equivalence of the Chinese and English versions. During the piloting of the questionnaire, Hong Kong teachers all felt that the translation was correct and appropriate.

Other than the difference in language, the three versions of the questionnaire for the three places only differed in terminology used in item 3 of Section A (Qualifications), and the school subjects that appeared in item 1 of Section B.

Observation Schedule

A brief review of the literature on classroom observation methodology shows that there are in general two different approaches to classroom observation: the "systematic observation" approach and the "ethnographic" approach. The first uses an observational system to reduce classroom behaviour to small-scale units under pre-determined categories (e.g. Flander's interaction analysis categories, 1970) suitable for tabulation and statistical analysis. This methodology is claimed to be "objective" in the sense that the observer is not required to make a lot of inferences during the recording process. Hence this process of recording data is termed "low-inference" recording, and with well-trained observers, high inter-rater reliability can be achieved.

The second approach on the other hand typically uses participant observation during which the observer "immerses" himself in the situation he is observing for a long duration, interacting with the subjects (called informants) and also formally interviewing them. During the whole process, detailed field notes on the focus of the study as well as on other background information (for example, the physical setting, the way different parties interact etc) are taken. Words of the "informants" are taken down in full and some of these words are quoted verbatim in the research report (see for example Delamont and Galton, 1986). The ethnographer claims to acknowledge the complexity of the classroom situation and uses a holistic framework, basing the observation not on pre-determined categories but according to the context of the teaching. Inevitably, the observer has to draw interpretations and make judgements even during the observation and recording

process. Hence this process of recording data is termed "high-inference" recording. But ethnographers claim that high validity would be achieved using this methodology.

The merits and limitations of the two approaches have been debated thoroughly elsewhere (see for example Delamont 1984, and McIntyre 1980) and will not be repeated here. It seemed to the researcher that systematic observation should only be used when the researcher, backed by well tested educational theories, was very sure about what exactly he was trying to observe. In systematic observation, the influence of other factors on the variable under observation is ignored during the observation process because if the observer takes other factors into consideration, he is making inferences and that affects the "objective" recording process. For example, many systematic observational studies measured the interaction between teachers and students, and the frequencies of the occurrence of certain behaviours such as instances when the teacher praised the students were counted. But the context under which those praises were given, and whether the observer felt that the praises were appropriate or not etc. were usually not recorded. So before the observations actually took place, the researcher had to convince himself that the mere frequencies of occurrence of these behaviours were significant factors in the education process.

For an ethnographic approach, the researcher can be less sure about what exactly the focus of the observation is at the beginning, and he tries to clarify what behaviours to look for as he gains experiences in his observation. He is less interested in quantifying behaviours, and can feel free to infer and try to get meaning out of what he is observing.

The two approaches discussed above are based on quite different research paradigms, but the present researcher does not think that their uses are mutually exclusive. However the nature of the present study made the use of systematic observation very difficult. Firstly, the contexts in the three places under study differed tremendously. Under different cultures and different circumstances, the same behaviour might express very different meanings in the three places. Secondly, there was still a lack of well developed theories attributing the different behaviours of people to different cultures. Thirdly, from the preliminary visits, it was found that the classroom practice in the three places differed significantly, so it would be difficult to devise a single systematic observation schedule that could capture the salient features of the practice in the three places in a quantifiable form³.

On the other hand, the use of a purely ethnographic approach did not seem feasible for the present study either, since one requirement of this approach was a prolonged immersion in the situations being studied. Time and financial constraints made an ethnographic study in all the three places so far away from each other not practicable. Also, it would be difficult to persuade a school in China to allow an outsider to stay in the school for a prolonged period to do research. (Only very atypical schools in China would allow such research to take place.)

³ For example, Flander's interaction analysis categories (1970) would not be suitable for this study because many schools in London followed individualized learning programmes and there were very little interaction between the teacher and the students.

Taking the above discussions into consideration, the present study aimed at collecting mainly qualitative data during classroom observation. But instead of prolonged immersion into a few schools in each place, slightly more schools in each place would be studied in less detail in order to get an overall idea of the kind of classroom teaching activities in the three places. Non-participant, overt observations would be employed, the object of the observation being the behaviour of the teacher. The behaviour of the students would only be recorded when it was judged to illuminate the teaching method used by the teacher.

A simple observation schedule that could be used to record the occurrence of instances under a list of categories of "sensitizing" concepts (rather than a mere quantitative measuring scale) was developed. Both the frequencies of occurrence and detailed descriptions of the instances under the sensitizing concepts were to be recorded in the schedule. The list of sensitizing concepts was derived from the preliminary visits and a review of the relevant literature (see chapter four), and included: alternative methods of solving the same problem; using one's own thought-out method rather than following the teacher's method; trial and error method of solution; memorization of mathematical results or processes; the use of rigorous mathematical languages; the application of mathematics in everyday life; the appreciation of mathematics as part of our culture and for its intrinsic worth or intrinsic beauty; and studying mathematics for the sake of examinations. Other behaviours outside the categories of sensitizing concepts but which were judged to be important would also be noted down.

In addition to general background data (date, name of the school and teacher, grade, number of students, topic of the lesson, and the starting and ending time), the schedule also allowed for the recording of the durations for different activities during the course of the lesson. These included: lecturing, questioning, class-work on an individual basis and on a group basis, maintaining discipline, giving administrative instructions, and silence. The amount of questioning was also to be recorded, and any differential treatment of boys and girls in the questioning could be noted. For class exercise, whether different students were expected or allowed to do different mathematics could be recorded, and the schedule also allowed for taking note of the audio-visual teaching aids used and the amount of homework assigned to students. A draft of the schedule is in Appendix D.

Interviews

Interviewing methods range from highly structured ones with the interviewee's responses coded according to pre-determined categories (rather like a verbally administered questionnaire) to open ended, exploratory conversations between the interviewer and the interviewee. For this study, the interviews with teachers were meant to gather their views on mathematics and mathematics education. Other interesting points that arose during the interviews would also be probed into further, especially if those were views that differed markedly from place to place. For this reason, semi-structured interviews would be used.

The original purpose of these interviews was to "triangulate" the interview results with the findings from the questionnaire and the classroom observations. Teachers would be asked during the interviews to express their views on the nature of mathematics, the teaching and learning of mathematics, the official syllabus (if there was one), the problems they encountered in teaching mathematics, and their concerns about mathematics education in general. The interview schedule is in Appendix F.

The Pilot Study

Studies were conducted in Beijing, Hong Kong and London in order to pilot the questionnaire, the classroom observation schedule and the semi-structured interviewing schedule that were to be used in the main study. The purpose of piloting the questionnaire was to see if the questions were unambiguous and meaningful to the teachers, and the purpose of piloting the observation schedule was to find out whether useful information on the teaching of mathematics in the three places could be recorded using the schedule. The interviews were conducted to identify any possible interviewing problems in the main study.

The pilot studies for Beijing, Hong Kong and London took place in December 1988, January to February 1989, and March 1989 respectively. Four schools with a variety of background participated in the studies in each of the cities. In each school, mathematics lessons were observed, and after the lessons, the teachers were asked to complete the

questionnaire and give their comments on the questionnaire. The same teachers were also interviewed based on the semi-structured interviewing schedule. Since the arrangement in each school differed, the number of lessons observed and the number of teachers interviewed were different for each school. As this was a piloting exercise, the arrangement was thought to be acceptable, but in order to get a sufficient number of completed questionnaires for analysis, teachers in a fifth school in each of Beijing and London were asked to complete the questionnaire as well. The particulars of the lessons observed and the number of completed questionnaire received are shown in table 3.1 below:

	Number of lessons observed (in 4 schools)	Copies of completed questionnaire
Beijing	5 (all are year 1 lessons)	13 (5 schools)
Hong Kong	11 (all are year 1 lessons)	10 (4 schools)
London	13 (7 year 1 lessons, 2 year 2 lessons and 4 year 3 lessons)	16 (5 schools)

table 3.1: Number of Lessons Observed and
Copies of Questionnaire Received in the Pilot Study

Pilot of the Questionnaire and the Interviews

The questionnaire took an average of about 14 minutes to complete (with a minimum of 8 minutes and a maximum of 26 minutes). In general teachers found the wording of the questionnaire unambiguous, but many teachers had difficulty in completing the section on workload. Some of the wording in the questions under this section was unclear, and also this section took up a very substantial proportion of the time for completing the questionnaire. After the pilot study, it was decided the information on workload, though

important, was not worth the trade-off of the amount of time used, and would be deleted from the questionnaire in the main study. Other alterations on the questionnaire include:

1. To avoid confusion arising from the different ways of responding to questions in Section B, this section was divided into two sections: the first four items requiring respondents to rank order became the new Section B, while the remaining items using a Likert scale became Section C.
2. Some teachers ticked instead of ranked order the first four items in Section B. In order to remind respondents to rank order, the instruction of rank ordering would be given at the stem of every item in addition to the instruction at the beginning of the Section.
3. The sections on Academic and Professional Qualifications (item 3 of section A) were combined, because some qualifications were considered both academic and professional (e.g. B.Ed., graduates of Colleges of Education in Hong Kong), and keeping two separate sections might be confusing.
4. Other alterations were minor changes in wording.

A tentative analysis of the results of the pilot questionnaires showed that the instrument seemed to be able to capture wide differences in views of teachers from the three places, and interesting and meaningful information seemed to have emerged. So with the revision mentioned above, the questionnaire was to be used in the main study. The three versions

(for teachers in Beijing, Hong Kong and London) of the revised questionnaire for use in the main study are given in Appendix C.

During the pilot study, many teachers told the researcher that they found the questions in the questionnaire very interesting. In Beijing, during the interviews with teachers after they had completed the questionnaire, some teachers went on very naturally to discuss the issues raised in the questionnaire. The researcher found these discussions very useful and felt that the questionnaire itself (instead of an interviewing schedule) could be used as the basis for the interviews with teachers (i.e. the teachers were asked to comment on the questionnaire rather than asked direct questions about their views on mathematics and mathematics teaching). In so doing, the interviews would become more natural and would help to a great extent overcome the reservedness of Beijing teachers mentioned in the section under the Preliminary Visits. Although teachers in London and Hong Kong were less reserved and did not need the questionnaire to help the interviewing, the researcher thought that the questionnaire could also be used as basis for the interviews in these two places as well.

During the pilot study in London, it was found that teachers there were very busy, and conducting long interviews with them could be difficult to arrange. So it was decided that the interviews with teachers should be short and concise, and only Section B and items 1 to 3 of Section C of the questionnaire (new version) would be discussed during the interviews. So for the main study, the "interviewing schedule" would consist of Section B and items 1 to 3 of Section C, plus questions asking teachers about their general problems and concerns in mathematics education.

Pilot of the Classroom Observation Schedule

During the pilot study in each of the three places, mathematics lessons were observed and recorded by the researcher himself. In addition to recording using the schedule in Appendix D, field notes concerning the physical setting of the school and the classroom, the development of the lessons (content, approach, examples used etc.) and the students' responses were taken during the observation or immediately after the lesson (the latest by the same evening of the day). In addition, the lessons were audio-taped.

As far as the observation schedule is concerned, the results from the pilot were not too satisfactory. It seemed that the only useful information gathered was the time spent on various activities in the lessons. Most of the categories under the sensitizing concepts were empty. The audio-recording went well for Beijing and Hong Kong. But in London, some schools had some or all of their lessons conducted using individualized learning programmes⁴, and for those lessons, very little was audio-recorded. The field notes in contrast proved to be more useful for collecting data on what happened in the classrooms.

As a result of piloting the observation schedule, it was decided not to use the categories of sensitizing concepts for recording data during the main study. The lessons in the main study would still be audio-taped, and detailed narrative field-notes on non-verbal aspects of the lessons would be taken. The sensitizing concepts will only be used during the data

⁴ In the four schools visited by the researcher, two used individualized learning for the first two years and whole-class teaching in year 3, one used individualized learning for all three levels, and one used whole-class teaching for all levels.

analysis stage (see chapter seven). The revised observation schedule therefore only allowed for the organisation of the teaching and the particulars of the lessons to be recorded, and a large space was left for taking field-notes. A revised schedule for use in the main study is in Appendix E.

The Main Study

Questionnaire

About 600 questionnaires were distributed in each of the three cities under study. For the convenience of administration, schools (instead of individual teachers) were sampled and all teachers in the sampled schools who taught junior secondary mathematics were asked to complete the questionnaire. Since the researcher had to rely on the Chinese government education authority to distribute the questionnaires in Beijing, and the education systems in the three places differed significantly, the sampling procedures in the three places were slightly different. How the schools were chosen and how the questionnaires were distributed and collected are described below.

The schools in Beijing were chosen by the Beijing Education Department after discussion between the researcher and the personnel responsible for mathematics education, and the schools were said to form a sample with a variety of background within Greater Beijing.

The questionnaires were sent out and collected through the Department between September and November 1989. They were distributed to the schools by officials in charge of secondary mathematics education in the Department when they were making their regular visits to the schools. Some of the completed questionnaires were collected by the officials at a subsequent visit, others were sent back by post using a stamped envelope provided. Altogether 600 questionnaires were sent, and there were 299 valid returns.

In Hong Kong and London, the questionnaires were distributed by the researcher himself. In Hong Kong, a list of schools was randomly drawn from a list of all secondary schools in Hong Kong. For those schools drawn which had teachers studying the in-service Post-graduate Certificate in Education course or other courses at the Faculty of Education in the University of Hong Kong where the researcher was working (of the 430 schools in Hong Kong, nearly 300 schools had teachers studying in the University), the teachers were asked to distribute the questionnaires to their colleagues who were teaching junior secondary mathematics. For the rest of the schools chosen, the heads of the schools were contacted by telephone and invited to participate in the research. If they agreed (most of them did), a number of questionnaires equal to the number of teachers teaching junior secondary mathematics in the school were sent to the school by post together with a stamped envelope. The schools in the list were used one by one in the manner described above until 600 questionnaires were sent out. The distribution and collection of the questionnaires took place between April and June 1990. Altogether there were 362 returns.

In London, for the maintained schools, the respective Local Educational Authorities (LEA) had to be approached first to get permission to administer the questionnaire (and also to conduct classroom observations) in their schools. Because of this practical constraint, it was decided to choose schools from within five boroughs of a variety of background instead of sampling from all schools within Greater London. The School Relations Office of the University of London Institute of Education helped to choose boroughs and approach the LEA concerned. Independent schools were approached directly by the researcher. Altogether there were 108 independent and maintained schools in those five boroughs, and a random representative (independent and maintained schools in each borough) subset of 86 schools was chosen from the 108 schools. Each chosen school was sent seven questionnaires together with a stamped envelope, giving a total of 602 questionnaires sent. The first batch of (602) questionnaires was sent out in early December 1990, and reminders and follow-up questionnaires were sent to those schools who did not respond to the first exercise by the end of January 1991. There were altogether 228 returns.

Classroom Observations

The classroom observations were done in December 1989 for Beijing, March to May 1990 for Hong Kong, and November 1990 to February 1991 for London. Six regions in each city were chosen, and three schools were visited in each region, making a total of 18 schools for each city.

School visits in Beijing were arranged by the personnel responsible for secondary education in the State Education Commission. Two regions in the inner city, two in the suburb, and two in the country side were chosen, and in each region, a "key-point" school (see chapter four), an "average" school and a "below average" school were visited, as requested by the researcher. All the schools were mixed (co-educational) schools.

In Hong Kong, schools were chosen from six out of the 19 administrative regions for study. Two of the chosen regions were in the New Territories (formerly rural areas in Hong Kong, but most of these areas have become urbanized in recent years), one in a more rural area and the other a new town. The third and fourth regions were relatively old regions, one more well off economically than the other. The fifth region was a residential region, and the last one an industrial region. A mixture of more established and newly founded schools were chosen for visit. Two out of the 18 schools were government-run schools, two were private schools, and the remaining 14 were government subsidized schools, roughly proportional to the number of these schools in the whole territory. Three of the schools were boys' schools, and another three were girls' schools, the rest being mixed schools. All the visits were arranged by the researcher himself directly with the schools.

In London, schools were chosen from six boroughs within Greater London. Two of the boroughs were from inner London and formerly belonged to Inner London Education Authority, the rest were from the outer boroughs. All the schools were chosen and approached by the School Relations Office of the University of London Institute of Education after discussion between the researcher and the person in charge in the Office,

and the boroughs were said to represent a spectrum of background among the boroughs in Greater London. Among the schools, three were independent schools (one each of boys, girls and mixed schools) and 15 were maintained schools (including one selected girls' school, one comprehensive girls' school, two comprehensive boys' schools, eight comprehensive mixed schools and three comprehensive middle schools).

The researcher requested observing two junior secondary mathematics lessons in each school. This was achieved exactly in Hong Kong. In Beijing, two lessons each were observed in 16 of the schools. Only one lesson was observed in one school but three lessons were observed in another, giving the same total of 36 lessons observed. In London, because of some slight misunderstanding in arrangement, one to four lessons were observed in each school, giving a total of 40 lessons observed. The distribution of the grades of lessons observed in the three places is shown in table 3.2 below:

	Year 1	Year 2	Year 3	Total
China	9	15	12	36
Hong Kong	13	12	11	36
London	16	12	12	40

table 3.2: Distribution of Grades of Lessons Observed in the Main Study

Interviews

Some difficulties were encountered when interviewing teachers in Beijing. As pointed out in the section on the preliminary visits, the original intention of the researcher was to conduct interviews individually with teachers. But the personnel in the State Education Commission who arranged the visits had arranged group interviews instead, despite previous requests of individual interviews by the researcher. After the first two visits, the researcher reiterated his desire to have interviews conducted individually. But the personnel responsible courteously denied the request, saying that teachers in China were more used to group discussions and that he wanted to save the researcher's time.

The limitations of group interviews in the Chinese setting were already pointed out in the section on the preliminary visits. But at this stage of the study, the researcher had to accept that this was a limitation of conducting educational research in China.

Another interview problem occurred in London. Teachers in London seemed very busy, and the recent implementation of the National Curriculum might have contributed partly to this. Despite requests by the researcher through the School Relations Office of the University of London Institute of Education to interview teachers, some teachers denied interviews because they were too busy. Other teachers could only spare very little time talking to the researcher, usually during recess time or talking over lunch, and often interrupted by students coming to see the teachers.

Because of these limitations in interviewing teachers, the study has to rely more on the other instruments for collecting information on teachers' views, and the interviews can only be used to supplement the information gathered by other instruments.

In this chapter, the methods of data collection, the development of the data collection instruments, the preliminary visits, the pilot study and the main study have been described and discussed in some detail. The researcher feels that there is a need for so doing, because without an understanding of the data collection process and its rationale, it would be difficult to judge whether the data analysis is adequate and the interpretation of results appropriate. Data analysis and discussion of the results of the main study will be presented in chapters five to eight, but before that, chapter four will first give an account of the contexts of the curricula in the three places under study.

Chapter 4: The Contexts of the Curriculum

Before presenting results of the comparison of the mathematics curriculum in China, Hong Kong and England in the coming chapters, it is essential that the contexts of the curriculum in the three places be described. As Bereday points out (1964:Preface, p.ix):

Educational facts are deeply enmeshed in a matrix of other social circumstance. They cannot be compared without a careful accounting for the total situation.

In chapter one, the relative cultural locations of the three places under study have already been mentioned briefly. In this chapter, some aspects of the Chinese culture in contrast with Western cultures and relevant to the present study will be briefly reviewed to provide the social and cultural setting for the comparison. Here the culture in England is assumed to be a typically Western one, and that in Hong Kong is influenced by both Chinese and Western cultures. So a review of literature on the Chinese culture in contrast with Western cultures will provide a background for a comparison involving the three places. Then the education systems will be briefly described and history of development of the curriculum in the three places will be traced to provide the system and historical contexts respectively.

The Chinese Culture in Contrast with Western Cultures

Stability of the Chinese Culture

The Chinese constitute over one fifth of the world's population. In addition to the over 1.1 billion of them in the People's Republic, overseas Chinese spread among countries along the Asian Pacific rim, in North American and in nearly all parts of the world. Despite these diverse social and political settings, it seems that the characteristics of the Chinese culture are relatively stable and consistent among them, as shown by large scale multicultural studies by Hofstede (1983) and the Chinese Culture Connection (1987).

One possible explanation of this stability of the Chinese character lies in its stress on the relationship between family (and clan) members and its reliance on the family to perpetuate its values, so that even when the political system changes, these traditional values are still preserved in the families (and clans). For the overseas Chinese, it is a common phenomenon that they tend to stay close to each other forming "China towns" with the family units staying intact and upholding a strong relationship between their members. In this way traditional Chinese characteristics are preserved for generations even when the family resides in a non-Chinese culture. It is this relatively stable and consistent character of the Chinese culture that Sun (1983:10) describes as a "super-stable structure".

The Social Orientation of the Chinese

Compared with Western cultures which stress independence and individualism (Taylor, 1987:235), the Chinese emphasize integration and harmony. Yang (1981:161) uses the term 'social orientation' (as opposed to individual orientation) to label this national character of the Chinese, and describes it as:

a tendency for a person to act in accordance with external expectations or social norms, rather than with internal wishes or personal integrity, so that he would be able to protect his social self and function as an integral part of the social network.

This is consistent with the observation by Sun (1983:11), who gives the Chinese definition of man in the following manner:

'Man' can only be defined in terms of his social relations -- he is the sum total of his social roles.

Related with this social orientation of the Chinese are characteristics such as compliance, obedience, respect for superiors and filial piety. Some scholars attributed these characters to the influence of the thoughts of Confucius (see for example Fung, 1948; Bond and Hwang, 1986), and Kahn (1979) used his so-called "Post-Confucian Hypothesis" to explain the economic success of Japan, Korea and the overseas Chinese communities. As far as education and mathematics learning are concerned, the relevant characteristics which educators used to explain different practices between the Chinese and the Westerners are reviewed below:

A. Compliance

Compliance can be defined as "a tendency to yield to others" (the Collins Dictionary¹), and the Chinese are known to have a tendency of complying with rules or orders more than Westerners. Related to this characteristic is a strong tendency among Chinese for uniformity (Sun 1983) and conformity (Bond and Hwang, 1986). In the area of mathematics teaching and learning, this perhaps manifests itself most clearly in the Mathematical Understanding of Taiwan Students Project (MUT) cited in chapter two of this thesis. In Lin's paper (1988) which compared some of the findings of the project with those of the CSMS study in the U.K., he pointed out that Taiwan students were much less susceptible to the so-called "child-method" which was so prevalent among British subjects of the CSMS study (see Booth, 1981). One piece of evidence that Lin used was the narrower facility spread in the Taiwan data. Similar results were obtained by the researcher in a replication study (for Algebra only) done in Hong Kong in 1988. In the Hong Kong study, the standard deviations (s.d.) of the facilities for four items with results in all the three places available were computed. The s.d. for England were found to be much larger than those in Taiwan and Hong Kong (Leung, 1989:128).

Lin attributed the different performance of the English and Taiwanese students to the different emphases of educators in the two places on mathematics learning, and quoted the Cockcroft Report to support his argument; but he did not discuss why there were

¹ Hanks, P. (ed.) (1990). *The Collins Paperback English Dictionary* (second edition). Glasgow: William Collins Sons & Co.

different emphases. The cultural differences in this area of compliance and uniformity may provide a plausible explanation to the different performance in mathematics.

B. Obedience, Respect for Superiors and Filial Piety

Liu (1986:78) described this aspect of the Chinese character as a behavioral rule that Chinese are taught to obey from childhood:

If your superiors are present, or indirectly involved, in any situation, then you are to respect and obey them.

Liu drew on the "word study" from school textbooks, newspaper and magazines by Liu, Chuang and Wang (1975) as evidence to support his proposed behavioral rule. If Liu's proposal is valid, then it perhaps helps to explain the relatively good discipline of students observed in classrooms influenced by Chinese culture (Wares and Becker, 1983; Leestma, Bennett and others, 1987; see also chapter seven of this thesis).

On the negative side, Liu and Hsu (1974) used this very characteristic to explain the poor performance of Taiwanese university students in a Chinese version of the Torrance test (Torrance 1966). The Chinese were found to be more rigid in thought and less original. In another study by Douglas and Wong (1977), Hong Kong pupils, especially female, were found to perform worse than their counter-parts in America on three Piagetian formal-operations tasks (Combination of Colours, Invisible Magnet and Projection of Shadows). The researchers explained these results by saying that the Hong Kong culture did not encourage an "active, searching mode of behaviour" (Douglas and Wong,

1977:692). These rigid aptitudes of the Chinese may be reflected in their attitudes to other areas, such as their attitudes towards mathematics and mathematics learning.

C. The Chinese Stress on Memorization and Practice

Liu (1986:80,82) again described these in terms of two more Chinese behavioral rules:

If the purpose is to acquire the knowledge contained in an article, then the best strategy is to memorize the article.

If the purpose is to acquire any new cognitive skill, then the best strategy is to practise repeatedly.

Liu drew on his own surveys (1984) on the practice of the Taiwanese teachers in requiring their students to memorize every chapter in the Chinese language textbooks as evidence for the "memorization" rule, and a cross-cultural survey by Stevenson and his associates (cited in Cunningham, 1984) on the amount of homework by American, Japanese and Taiwanese grade one pupils as evidence for the "practice" rule.

Researchers have reported in the literature on the seemingly superior performance of Chinese subjects on memorization tasks. Hoosain (1979) for example found that the mean digit-span for Hong Kong undergraduates was about 2 digits higher than that reported for comparable samples in the West, and other findings pointed to the same conclusion (e.g. Huang and Liu, 1978). These results could have been the consequences of the "memorization" behavioral rule proposed by Liu.

D. The high expectations on student achievement

Chinese parents and teachers are known to attach great importance to the education and achievement of their children and students (see for example Sollenberger, 1968). The parents' expectations are shown in their child-rearing attitudes and practices (Ho and Kang, 1984), and the high expectations of students are manifested in the more demanding Chinese and Japanese mathematics textbooks (Stevenson, 85; Stigler and Perry, 1988b; Wares and Becker, 1983). This contrasts sharply with the attitudes in the West (e.g. the U.S., see Stevenson, 1987).

These high expectations may be related to the Chinese and Japanese attribution of academic success and failure. Stevenson, Lee and Stigler (1986), for example, found that "U.S. mothers are significantly more likely than Japanese mothers to believe that innate ability (as opposed to effort) underlies children's success in mathematics". Hess, Chang and McDevitt (1987), in a study comparing the attitudes of mothers and children from Chinese, Chinese-American and Caucasian-American families, obtained similar results. They found that the American mothers were more likely to attribute their children's failure to the lack of ability, while Chinese mothers usually attributed the failure to lack of effort and the success to the teaching in the schools. In the same study, Chinese mothers were also found to use punishment and threaten punishment more often than the American mothers, who provided "rewards without setting higher standards" (Hess, Chang and McDevitt, 1987:185) more often. In another study, Stevenson (1987) commented that an Asian attitude was that "differences in innate ability are de-emphasized, and the potential for change throughout life is believed to lie within the individual. As a result,

teachers and parents believe that any child is capable of learning. ... that any student can be good at mathematics if he or she works hard enough" (Stevenson, 1987:8). In the terminology of the Attribution theorists, Chinese parents and teachers are more likely to attribute their children's success and failure to internal and controllable factors. These attitudes are communicated to their children through feedback/reactions that in turn cause their children to put more effort into their study.

E. The attitude towards study

The traditional Chinese attitude towards studying and learning is that it is a hardship. One should pay the price of diligence and perseverance in order to succeed, and is not supposed to "enjoy" the studying (Garvey and Jackson, 1975). Every Chinese is familiar with the many ancient Chinese folk stories about famous figures having had a hard time studying and eventually becoming successful (see for example Huang, 1969). In the report by Stevenson (1987) quoted above, this point was well recognised. Stevenson commented that "Asian parents teach their children early that the route to success lies in hard work" (see also Hess, Chang and McDevitt, 1987). But we should not generalize by simply saying that Asians are relying solely on extrinsic incentives to motivate their children for academic success, for the distinction between intrinsic and extrinsic motivations is not clear-cut and may be complementary (Lynn, 1988, chapters five and six).

The Chinese Classroom in Contrast with Classrooms in the West

Very few studies relate classroom practice to differences in the cultural setting. One exception is the University of Michigan Studies cited in chapter two of this thesis. In one of the articles published by the director of the Studies (Stevenson, 1987), the following classroom practices were listed as reflection of the Chinese and Japanese cultural philosophies:

In Taiwan and Japan,

1. more time is devoted to mathematics teaching,
2. there is more direct instruction during mathematics lessons,
3. mathematics is typically taught in the morning, punctuated with frequent recesses (this contributes to children's ability to concentrate and teachers' "spirited instructional modes"), and
4. teachers are required to be in charge of the classroom for fewer hours (than are American teachers).

Stevenson might be too optimistic about the mathematics education in Taiwan and Japan and too critical of the mathematics education in America, but his work draws educators' attention to the cultural influence on educational practice in general and the teaching and learning of mathematics in particular.

When comparing the mathematics curricula between China, Hong Kong and England in the present study, the cultural and societal factors reviewed above as well as curricular

factors will be drawn upon to suggest possible reasons for any differences found in the results of the analyses.

The Education System in China, England and Hong Kong

China

China is a vast country occupying a land of 9.6 million square kilometres with a huge population of 1.16 billions, 86% of which live in rural areas. There are about 56 nationalities, though an overwhelming majority (94%) of the population belong to the nationality of Han. Providing education for the enormous number of school-aged children distributed in diverse localities is no easy task. Legislations were enacted in 1982 and 1985 for compulsory primary and junior secondary education respectively. These were to be achieved in phases for different parts of the country according to the local situations. At present, universal education up to junior secondary level is basically achieved in large cities, but in some remote areas, even meeting the target of universal primary education proves to be difficult.

Overall responsibility for education rests with the State Education Commission (SEdC) set up in 1985 to replace the former Ministry of Education. The SEdC, under the leadership of a vice premier, formulates education policies, conducts educational planning,

issues regulations governing the running of educational institutions, and oversees the execution of policies. Education bureaux and departments are set up in the 30 provinces, municipalities and autonomous regions to execute the policies laid down by SEdC and supervise the running of schools in their local areas.

Secondary education in China is provided in two stages: junior secondary and senior secondary. Junior secondary education provides basic knowledge and is part of the nine years of compulsory education. Senior secondary education is in general provided in regular or grammar schools, and about 35% of the students study in agricultural and vocational/technical schools. Because of the scarcity in resources, the Chinese adopt a system of "key-point" schools whereby special resources are allocated to some schools which select the best students through public examinations for education. These schools are "the best of the best, favoured in facilities, teaching staff, and in funding" (Unger, 1982:19). The disparity in facilities between these key-point schools and other schools is very considerable. This elitist system is "justified in terms of the need to identify and train China's most talented youth in order to speed up the process of modernization, thus eventually benefiting the entire society" (Hawkins, 1983:35). All Chinese schools are mixed (co-educational) schools.

Formal education starts at the age of six or seven, although some children would have spent two to three years in nurseries or kindergartens before entering primary school. After five or six years of primary education, students take a provincial or municipal public examination to compete for entry into key-point and other secondary schools. Junior secondary schooling lasts three or four years, and another provincial or municipal

examination is held to decide who can go on to senior secondary schools. A nation-wide examination after the senior secondary stage selects students for higher education, though there have been some recent efforts in trying to set examination papers at provincial rather than national level.

The curricula in various subjects are centrally devised in committees set up under SEdC. In the past, textbooks were produced solely by the People's Education Press, the official publisher of educational materials in the country. Recently, the government appointed publishers in some provinces to publish textbooks that suit the need of their localities. But all textbooks are to adhere to the national syllabuses and have to be scrutinised by a central committee under SEdC. Subjects studied at junior secondary level include: Chinese, mathematics, foreign languages, politics, history, geography, biology, physics, chemistry, physiology and hygiene, physical education, music, and fine arts.

The subject of mathematics is an important component in the overall curriculum. Of the 3168 teaching hours in the three years of junior secondary education, 486 hours were allocated to mathematics, comprising 14.8% of the time allocation, second in emphasis only to the study of (the Chinese) language².

The Beijing metropolitan area consists of 4 inner city regions, 4 suburb regions, and 10 rural counties, and has a population of 10 million. Altogether there are about 700 secondary schools in the metropolitan area.

² Zhongguo Jiaoyu Bao, 7 November 1991, p.3

England

England is one of the four constituent countries of the United Kingdom. With a population of 47.3 million, it is the most populous among the four (total population in U.K. is 56.8 million). About 4.4% of the English population are ethnic minorities, 41% of whom live in the Greater London area.

England (and Wales) is said to have a centralised education system which is locally administered. One of the system's distinctive features "is the large degree to which responsibility for provision (of education) is decentralised" (Central Office of Information, 1990:176). Overall responsibility for all aspects of education rests with the Secretary of State for Education and Science. The Department of Education and Science (DES) formulates educational policies, allocates resources, and influences the other partners in the education service. The Her Majesty's Inspectors (HMI) are an independent group attached to the DES who report to the Secretary of State on the education provided in schools and also give professional advice to the DES. The provision of maintained (i.e. publicly funded) school education has traditionally been the responsibility of local education authorities (LEAs), which are funded by central government and local taxes, while day to day running of the schools is managed by the governing bodies and the head teachers. But as a result of the 1988 Education Reform Act, some of the functions performed by LEAs are relocated to central government and the governing bodies of schools.

About 90% of the schools are maintained schools, and just under one third of these are voluntary schools run by religious denominations (though still mainly funded publicly). The rest of the maintained schools are county schools run by LEAs. Over 90% of the maintained schools are comprehensive schools, admitting students of different abilities without selection. The rest are grammar and secondary modern schools, catering for high and low academic ability students respectively. Most of the maintained schools are mixed (co-educational) schools. The independent or private schools are outside the publicly maintained sector, but they must register with the appropriate education department and are open to inspection (Central Office of Information, 1990:179). They are usually selective, grammar schools and most of the secondary independent schools are single-sex as well.

Compulsory education between the ages of 5 and 16 has been enforced from 1973. Children usually spend two years in nurseries or nursery schools before entering primary schools at the age of five. Primary education usually lasts six years, after which students are transferred to secondary schools for five more years of compulsory education. There is also a small proportion of middle schools which take students between the ages of 8 to 12, 9 to 13, or 10 to 14. Before 1991, there was no public examination until the end of secondary education, when most of the students took the General Certificate of Secondary Education (GCSE) examination, a replacement of the former General Certificate of Education (GCE) Ordinary (O) level and the Certificate of Secondary Education (CSE) examinations in 1988. A minority of students entered for other examinations or followed assessment schemes certified by examination boards such as the Oxford and Cambridge Schools Examination Board. The 1988 Education Reform Act

introduced regular assessment of performance at the ages of 7, 11, 14 and 16, and the testing at age 7 started in 1991.

Before 1988, schools had almost total freedom in deciding the curriculum they offered. The only requirement by law was for religious education to be provided. In practice, the influence of public examinations (GCSE and the former GCE and CSE) in the senior secondary years and the backwash effects on the lower secondary years shaped the school curriculum to some extent. The 1988 Education Reform Act prescribed a National Curriculum for maintained schools which includes the core subjects of English, Mathematics and Science as well as the other foundation subjects of History, Geography, Technology, Music, Art, Physical Education and a modern foreign language. Although time allocation for these subjects was not prescribed, it was suggested that about 10% of the overall teaching time should be allocated to mathematics (DES, 1987:7)³.

The London metropolitan area has a population of 6.7 million, and before 1988 education was administered under 20 Greater London boroughs and the Inner London Education Authority (ILEA). After 1988, ILEA was dissolved and its functions relocated to 13 LEAs, giving a total of 33 LEAs in metropolitan London at present. There are a total of about 570 secondary schools (of which about 20% are independent schools) in the metropolitan area.

³. Although the title of the table on p.7 says "SECONDARY PHASE - YEARS 4 AND 5 ALLOCATION OF CURRICULUM TIME", paragraph 15 (p.6) of the same document indicated the proposal was for the whole duration of "compulsory secondary schooling".

Hong Kong

Hong Kong came under the British rule in 1842 and since then has inherited to a large extent the British education system. The Colony is one of the most densely populated areas in the world (5.7 million people in an area of 1000 square kilometres, less than 20% of which are built up areas). The population is homogeneously Chinese, 80% of which speak Cantonese, the dialect of Guangzhou (Canton). There is a minority (about 2%) of non-Chinese in the population, mostly Europeans, and their children are catered for by a separate education system. Education for children between the ages 6 and 15 has been made compulsory since 1978.

The territory of Hong Kong is divided into 19 zones for the purpose of administration. According to the Education Ordinance, the Director of Education is "charged with the superintendence of matters relating to education in Hong Kong", and his responsibilities include promoting the education of the people of Hong Kong and controlling and directing education policy. The Director of Education heads the Education Department, and divisional officers of the Department carry out duties in administration and control of the education system. Schools are managed by management committees set up by the sponsoring bodies of the schools, though all schools are registered under and supervised by the Education Department.

There are altogether about 430 secondary schools in the 19 zones. In terms of mode of financing, these schools are either government-operated (9%), government-subsidized

(72%) or privately run but under the monitor of the Education Department (19%). In terms of the curriculum, there are four types of schools: Anglo-Chinese secondary schools, Chinese middle schools, secondary technical schools and prevocational schools. The first two are grammar schools, but the distinction between these and the technical schools is not very marked since technical schools also follow a rather academic curriculum, and many grammar schools include some practical subjects in their curriculum. In prevocational schools, 40-45% of the curriculum is devoted to practical and technical subjects, but the students follow the same mathematics syllabus as the grammar school students. Theoretically, English is the medium of instruction in all the schools except the Chinese middle schools which use Chinese as the medium. But since the popularization of education, the English standard of students in most schools is not high enough for them to be able to follow instructions totally in English, and so although English textbooks are still used in these schools, the teaching is predominantly done in Chinese, switching to English only when technical terms are encountered. So as far as the junior secondary level is concerned, the mathematics curricula in all four types of schools do not differ much.

Children enter primary schools at the age of six, most of them having spent two or three years in kindergartens, which are all privately run. After six years of primary education, students are assigned to different secondary schools by means of a mechanism called the Secondary School Placement Allocation (SSPA) Scheme, which scales students' internal assessment in schools with performance in a public Academic Aptitude Test. The students proceed for three more years of compulsory education before they are allocated again for senior secondary study, although the majority of students stay on in the same

school. At the end of secondary education, students take the Hong Kong Certificate of Education Examination (HKCEE), and schools admit students to their sixth form according to students' performances in this examination. Two more years of sixth forms lead on to the Hong Kong Advanced Level Examination, and students are selected for higher education on their results in this examination. In 1990, about 89% of the cohort moved on to senior secondary education, 25% went on to sixth forms, and only about 6% got a place in higher education locally.

Subjects offered at the junior secondary level include: Chinese, English, Mathematics, Science, Social Studies, Chinese History, Physical Education, Music, Ethical or Religious Education, and practical subjects such as Art and Design, Home Economics and Design and Technology.

Mathematics occupies a significant place in the overall curriculum. Passing in mathematics at the HKCEE level is a pre-requisite for entry into the universities in Hong Kong as well as for some employment. In a typical junior secondary time-table, mathematics occupies five 40-minute periods (out of a total of 35 to 40 periods, see Curriculum Development Committee Hong Kong, 1975a:11) per week, second in emphasis only to the subject of English Language.

Summary

From the descriptions above, it can be seen that China is an immense country geographically and in terms of its population. There is a very high degree of centralization of control in its huge education system combined with a high expectation of ideological conformity within the society as a whole and within the teaching profession. Resources in education are relatively scarce, and a substantial proportion of the teaching force is underqualified (see chapter six).

England is much smaller than China, but much bigger than Hong Kong in size and population. Education is traditionally very decentralized, but is currently moving towards greater centralization. There are more resources allocated to education when compared with China and Hong Kong, and the teaching force is relatively very well qualified (see chapter six).

Hong Kong is minute in scale compared with China and England geographically and in terms of its population. Although other aspects of its education system are very much influenced by the British system, the system itself, like that in China, is highly centralized.

From this summary, it is obvious that the three places under study are drastically different from each other, and their education systems are also very different. These differences have to be borne in mind in the comparison presented in the coming chapters.

A Brief History of Development of the Mathematics Curriculum in China, England and Hong Kong

A description of the history of development of the curriculum is essential, as it sets the context for the curriculum in the three places to be understood. As Eckstein *et al* (1982:9) point out, "in all comparative study it is important to recognise ... 'the inherited context', ... the precedents established by a nation's historical and cultural traditions". It is only when curricula are understood in their historical context that they can be compared meaningfully. What is attempted below is a brief description of the history of development of the mathematics curriculum in China, England and Hong Kong.

China

1. Before 1949

China is one of the oldest countries where mathematics as a discipline developed. Western mathematics began to influence China after the visit of Matteo Ricci in 1581; and in 1606, Ricci, with the help of Chu Kwong Kai, translated the first six chapters of Euclid's *Elements* into Chinese. Formal education in China began in 1862 in the late Ching Dynasty. At that time, schooling was for eight years, and mathematics was studied in the fourth, fifth and sixth years (Introduction to Mathematics, Algebra in the 4th year; *Elements* and trigonometry in the 5th year; Calculus in the 6th year).

In the 1930s and 1940s, the mathematics syllabus was very much influenced by Japan and the United States of America. Many of the textbooks used were either translated or adapted from foreign texts, but textbooks written by local mathematicians also began to appear. In junior secondary schools, the mathematics syllabus consisted of Arithmetic, Algebra, and Plane Geometry.

2. 1949-1958 The Transition Stage and Full Scale Learning from the USSR

After the founding of the People's Republic, education was more tightly controlled nationally. The overall educational aims in this period were the popularization of education and the development of the whole person. Between 1949 and 1952, a brief national outline for the mathematics syllabus governed the curriculum in the schools. But the mathematics content did not differ much from that of the pre-1949 era, except for some slight reduction in content at the senior secondary level.

The years 1952 to 1958 were a period of "full-scale learning" from the USSR. The first national curriculum in mathematics was announced in 1952, based on the curriculum in the USSR. Not only were the syllabuses translations of the Soviet syllabuses, but the textbooks, reference books for teachers, and the teaching methodology were all "imported" from the USSR. The emphases of Soviet mathematics education in this period were on basic knowledge and concepts, and on drill and practice, but applications of mathematics were de-emphasized. In particular, applications in commerce were totally abandoned, as these were considered characteristics of capitalistic societies. All these characteristics were transplanted to the 1952 Chinese curriculum.

3. 1958-65 Reform and Consolidation

During the "Great Leap Forward" in 1958, Chinese educators called for reform of the secondary curriculum. The system of 12 year education (6 years of primary and 6 years of secondary) was changed to a 9 or 10 year combined primary and secondary system. The existing curriculum was criticized as out-of-date, disconnected and unrelated to practice. The slogan in this period was "to learn more, to learn it quicker, to learn it better and to learn it more economically", and the target was to "overtake Britain and the United States".

In the secondary mathematics curriculum, radical changes were suggested and experimented. Euclidean Geometry was ousted from the syllabus, and topics such as Calculus, Differential Equations, Probability and Statistics were introduced. It was also suggested to use the concept of function as the basis to "unite numbers and forms, theory and practice, concepts and skills". Topics were not to be repeated in the curriculum, and many former junior secondary school topics (Negative Numbers, the use of the unknown 'x', plane geometry) were pushed down to the primary years. The mathematics content was to be more related to the needs of the "modern" society, and various provinces in the country started to prepare their own textbooks for experimentation.

By 1960, some educators felt that the reforms had gone too far. 1960-62 was a period of reflection and re-thinking. In the People's Congress in 1960, the Minister of Education Lu Ting-yi proposed that reforms should be "appropriate": the period of study should be shortened "appropriately", the standard should be raised "appropriately", the number of

hours of study should be controlled "appropriately" and Labour should be increased "appropriately". The proposal was meant to rectify some of the "inappropriateness" that appeared in the reforms of the past two years. This was also a period when a lot of "experiments" were carried out, and a number of "experimental" schools were set up in Beijing. It was believed that educational reforms should be based on well-tested experiments.

By 1962, it was in general agreed that the Great Leap Forward was too great a leap, and adjustments were needed. The number of years of education was switched back to 10 to 12 years. In mathematics, a new national curriculum was published in 1963. Euclidean Geometry was brought back to the secondary curriculum, and Analytic Geometry and some Probability and Statistics were added in. Algebra content was reduced, but the notion of function was still stressed, and Calculus was deleted from the syllabus. The textbooks for this syllabus were published between 1963 and 1965, and the reformed textbooks produced during the Great Leap Forward were abandoned.

The 1963 syllabus and textbooks were received favourably among mathematics educators. The Soviet influence on this syllabus was still considerable, but the syllabus seemed to have captured the strength of the Soviet tradition as well as taken into consideration the experiences in Western countries and the unique situation of China. Unfortunately, only after the syllabus and the textbooks had been used for a very short period of time, the Cultural Revolution broke out, and this syllabus and its textbooks were abandoned.

4. Cultural Revolution 1966-76

The Cultural Revolution was a period of turmoil for China. Regular schooling was severely interrupted and education was "greatly damaged". The development in the past 17 years was totally refuted, and utility was stressed at the expense of academic study. For example, in Physics teaching, the whole secondary school syllabus consisted of only the study of "three machines and one pump" that were commonly used in production. The People's Education Press, the traditional publishing arm of the Ministry of Education, was de-established, and various provinces produced their own textbooks according to the local situations and needs. In the realm of mathematics, topics such as book-keeping, mensuration and graph-plotting were greatly stressed. "Bringing out mathematics content through typical products" was the slogan, and topics such as "The application of fertilizers and quadratic equations" were found in a textbook published in the Guangdong area during this period.

5. 1976-present

After the fall of the "Gang of Four" in 1976, China began to recover from the "ten years of turmoil". The overall direction of the country was the "Four Modernizations", and education was to prepare students to fulfil this national goal. There was a strong feeling that the country had lost a few generations of personnel in the ten years, and there was a great need to produce more personnel in a shorter period of time. Also, as China began to open its door to Western countries, Western educational ideas began to influence the construction of education and the curriculum in China.

In the realm of mathematics education, experienced educators were commissioned by the government to produce a new curriculum and new textbooks. The slogan during this period was to "select and condense, increase and infiltrate": select and condense from the traditional mathematics content, increase or add in modern content (e.g. Calculus, Boolean Algebra, Probability and Statistics, Introduction to Computers etc. were added into the curriculum), and new ideas and methodologies were infiltrated into the curriculum (e.g. the idea and language of Sets). There was also suggestion of a unified mathematics text, combining the different branches of mathematics (Algebra, Geometry etc.).

The curriculum that resulted represented a sharp swing back to academic study of mathematics from the extreme emphasis on practice during the Cultural Revolution, but it was soon realized that the new curriculum was too demanding for the average student in China. The content was too difficult and abstract and the practical aspects of mathematics were not stressed enough. It was estimated that only about 25% of the student population (and secondary education was far from universal) were able to follow the new curriculum (interview with Su, 22 August 1988). Moreover, the authorities had underestimated the effect of the Cultural Revolution. There were simply not enough teachers who were competent enough to teach the new curriculum. Most schools were badly equipped and many teachers were unqualified. In particular, many teachers were unable to cope with the unified text. They were used to teaching a certain branch (say Algebra) of mathematics only and felt incompetent to teach the rest of the syllabus.

In 1980, it was decided that adjustment was again needed. Slightly different mathematics curricula were designed for Science and Arts students, and the curricula were reverted

back to being separated into the branches of Algebra, Geometry etc. Some modern and more difficult topics such as Boolean Algebra were deleted, but Calculus and Sets still remained in the syllabus.

In 1983 and 1985, the mathematics curricula at senior and junior secondary levels respectively were slightly diversified to suit the needs of students of different abilities. Each curriculum was divided into two different levels, one called "basic requirements" and the other "higher requirements". The basic requirements syllabus was meant for the average student and was derived from deleting the more difficult topics from the 1977 syllabus (for example, Calculus was deleted). The 1977 syllabus became the "higher requirements" and was meant for the more able students, especially those studying in key point schools. Two different sets of textbooks at the senior secondary level were published for these two levels by the People's Education Press. In junior secondary schools, the same set of textbooks was used for both requirements.

In May 1985, the Chinese government announced its decision to provide nine years of compulsory education (i.e. up to junior secondary level) for all. Because of the vast contrast of conditions in various parts of the country, different areas were allowed to achieve this target at different paces, but the majority of the children were to attain this goal by 1990. After the announcement of this decision on universal education, educators were commissioned to revise the syllabuses and produce textbooks to suit the need of the majority of the student population. It was decided that the requirement of the new syllabuses should in general be lower than that of the syllabuses at the time; there should be more flexibility, more choices for teachers and students, and there should be more

topics of a practical nature. It was also decided that although there would still be only one syllabus in the country, different textbooks would be published to suit the situations in different provinces and different schools within a province. In mathematics, in addition to textbooks that were to be published by the People's Education Press, the State Education Commission commissioned five to six more groups of educators to produce textbooks for use in 1990, and a committee was set up to scrutinize these textbooks. It was soon realized that the 1990 deadline was unreasonable, and so a decision was made to delay the publication of the new textbooks to 1992.

Meanwhile, it was felt that the existing syllabus and textbooks, even at the basic requirements level, were too abstract and demanding for the average child. Instead of waiting for the 1990 syllabus and the new textbooks, a transitional syllabus was devised in 1986 as an interim measure. This transitional syllabus was based on the former "Basic Requirements", but some topics were deleted, treated as optional or moved to a higher grade. The Higher Requirements topics were put in an appendix and were meant to be optional only. The basic idea was to lower the requirement and give teachers more flexibility in choosing the mathematics content in their teaching. This transitional syllabus was the syllabus that Chinese students were following when this study was conducted.

England

1. 1940's and 1950's

The 1944 Education Act is considered by many as one of the most important education acts passed in the country. It "replaced almost all previous education legislation and laid the foundation for the modern education system" (Statham *et al*, 1989:43). It set up, among other things, a unified system of free, compulsory schooling for all from the age of 5 to 15. This "rationalised" the existing 'tripartite' school system (Griffiths and Howson, 1974:56), with the grammar schools catered for the academically oriented, the technical schools for those whose "interests and abilities lie markedly in the field of applied science or applied art" (HMSO, 1941), and the secondary modern schools for the majority of children. Selection for entry into these schools was through an 11 plus examination. The curricula of these three types of schools were determined respectively by "tradition, vocational needs and default" (Howson, 1978: 183). The mathematics curriculum in grammar schools was geared towards School Certificate Examinations, and that in secondary modern schools was "grounded in elementary arithmetical tradition" (Cooper, 1985:47) and was "fairly formal and rather dull" (Cornelius, 1985:31).

As far as mathematics education is concerned, a potentially significant event which also happened in 1944 was the publication of the Jeffery Report by a group set up after a conference in 1943 of representatives from examining bodies, teachers' associations and the Mathematical Association. The report proposed changes in the syllabus for examinations taken at the age of 16. It suggested the fusion of mathematical subjects and

in particular a closer association of geometry and trigonometry. "Mixed" examinations (rather than separate papers on Geometry, Arithmetic and Algebra) and complete freedom in choice of method were recommended. It suggested bringing "mathematics more closely into relation with the life and experience of the pupil", and time for inclusion of more applications was to be obtained by drastic reductions in formal geometry and heavy calculations and manipulations in arithmetic and algebra.

Despite the suggestions of the Jeffery Report and the formation in the early fifties of the Association for Teaching Aids in Mathematics (renamed in 1962 as the Association of Teachers of Mathematics (ATM)) which advocated new approaches in teaching mathematics, "secondary school mathematics curricula had remained unchanged in respect of content and pedagogy from the first world war to the late 1950s" (Cooper, 1985:35).

2. 1960's

Drastic changes in mathematics curriculum came in the sixties, changes which are sometimes known as the modern mathematics movement. Cornelius considered the Southampton Mathematical Conference in 1961, which succeeded two earlier conferences in Oxford (1957) and Liverpool (1959), the "real beginning of 'modern' mathematics in England" (Cornelius, 1985:33). Various projects (including the School Mathematics Project (SMP)) were initiated to reform the mathematics curriculum. By this time, the "new math" movement in U.S. was already underway, and the American influence was strong (Cornelius, 1985:33), though the English projects kept "the tradition of applied mathematics and avoided the 'new math' excesses of overabstraction" (Howson,

1978:188). There were great changes in both content and teaching approach. New content such as Probability and Statistics, Sets and Logic, Matrices, and Algebraic Structures were introduced early in the secondary school curriculum. In approach, modern mathematics stressed the applications of mathematics, the unified nature of mathematics and rigour and precision in mathematical language.

In the educational scene at large, a motion was passed in 1965 in the House of Commons urging local authorities to "reorganize secondary education on comprehensive lines", and later in the year, the DES requested LEAs to "prepare and submit ... plans for reorganizing secondary education in their areas on comprehensive lines" (Griffiths and Howson, 1974:57). In the same year, the Certificate of Secondary Education (CSE) examination was introduced to provide a public examination at 16 plus for those of "average ability" and for whom the GCE was too demanding. Comprehensive education brought new problems to mathematics teaching in the schools. The response to the problems was a second wave of projects, the most noticeable being the Mathematics for the Majority Project developed by the Schools Council set up in 1964 and the Secondary Mathematics Individualised Learning Experiment (SMILE) developed by the Inner London Education Authority (ILEA). Both projects aimed at catering for the needs of students of "average and below average ability".

So the 1960s is a decade of reforms in mathematics curriculum in response to both the world-wide modern mathematics movement as well as the movement in the country towards comprehensive schooling. The reforms "left many teachers, parents and pupils in a state of bewilderment", and so "the 1960's ended in confusion" (Cornelius, 1985: 33).

3. 1970's

By the 1970s, the drawbacks of the modern mathematics movement began to surface. Teachers and parents found their children unable to cope with the abstraction and rigour of the new curriculum, and their basic mathematics skills were declining. As mathematics educators began to evaluate the changes of the previous decade, "the pendulum of reform began to swing back sharply and the mid-1970s witnessed a 'back to basics' movement" (Cornelius, 1985:33-34). Meanwhile, the efforts to produce individualized learning materials and materials for students of average and below average ability which had started in the late sixties continued. The decade also saw the advent of cheap calculators and computers. This compounded the problems faced by mathematics teachers, and it was against this background that the Cockcroft Committee was set up in 1978 to "consider the teaching of mathematics in primary and secondary schools in England and Wales, ... and to make recommendations" (Cockcroft, 1982:ix).

4. 1980's

The Cockcroft report was published in 1982. It is a comprehensive report which made comments on the teaching of mathematics as well as the impact of calculators and computers, examinations and assessment, facilities for teaching mathematics, and the supply, training and in-service support of teachers. The Cockcroft report was generally welcomed and "there was wide-spread acceptance of the report and little adverse comment" (Cornelius, 1985:28). It was considered by many as the most important report on mathematics education in recent years.

On teaching methods, the report recommended the use of a variety of approaches, including emphasis on problem solving and investigational work (para. 243, see also chapter five of this thesis). On content, it criticized the current syllabuses followed by the majority of pupils "as not suited to their level of attainment" because they were "constructed by using as starting points syllabuses designed for pupils in the top quarter of the range of attainment in mathematics" (para. 449 and 450). It suggested a "from the bottom upward" approach in developing syllabuses and went on to produce a "foundation list of mathematical topics" (para. 458) which "should form part of the mathematics syllabus for all pupils" (para. 455).

The recommendation of the Cockcroft report on assessment was one important contributing factor for the introduction in 1986 of the General Certificate in Secondary Education (GCSE) examination which replaced both the GCE O level and the CSE examinations. The first GCSE examination took place in the summer of 1988. In addition to unifying the two 16 plus examinations, "a major innovation of the GCSE is the production of national criteria governing all syllabuses" (Open University, 1986:5). General criteria set out the framework for all syllabuses and examinations, and specific criteria lay down requirements for individual subjects, including Mathematics. Many people welcomed this idea of a common examination at 16 plus, but some had reservations and saw this as the government's attempt to centralized its control over the curriculum which had up to now been in the hands of teachers.

The Education Reform Act in 1988 was seen by many as a further step by the government towards centralized control of the curriculum. The Act empowers the Secretary of State

to prescribe a common curriculum (called the National Curriculum) for pupils of compulsory school age in maintained schools, to set attainment targets for each of the constituent subjects of the National Curriculum at the ages of 7, 11, 14 and 16, and to make arrangements for assessing whether the attainment targets are met. The National Curriculum for Mathematics contains 296 statements of attainment in 14 attainment targets at ten levels, and implementation of the Curriculum started in 1989-90. Assessment for age 7 started in 1990-91, and that for age 14 in 1991-92.

Reactions to the National Curriculum varied. Some teachers and parents welcomed the move towards a common, broad and balanced curriculum with clear objectives set, while others considered the prescription of a unified curriculum instead of leaving it to the hands of teachers who know their students best a big step backward. In the realm of mathematics, some welcomed the emphases in the National Curriculum on the applications of mathematics, on investigation and on contemporary needs (Brown, 1990:35) while there were those who argued that "the National Curriculum has effectively emptied the mathematics curriculum of mathematics" (Dowling and Noss, 1990:1). Most teachers, however, were weary of the extra amount of administrative work that was needed to cope with the National Curriculum.

Hong Kong

1. 1940's and 1950's

Before the sixties, the mathematics syllabus was based upon pre-war models, with arithmetic, algebra, geometry and trigonometry being four separate areas of study within mathematics. There was no official teaching syllabus or curriculum guide. The only document that informed the curriculum in the classroom was the Hong Kong Certificate of Education Examination syllabus published by the Examinations Section of the Education Department, and it did influence greatly the teaching in the Hong Kong classrooms, where "great emphasis was laid upon computation and upon recognising 'typical' problems in examinations, an emphasis which often led to rote learning and excessive drill." (Brimer and Griffin, 1985:6)

2. 1960's

The ripples of the modern mathematics movement from the West were first felt in the early sixties. In the summer of 1962, a seminar was organized at the University of Hong Kong by a group of progressive mathematics educators to introduce to local teachers the new ideas and new trends in mathematics education in Western countries. As a result, a working group was set up to prepare modern mathematics textbooks for trial use in some local schools. Modelling on the SMP series in England and other modern mathematics projects in Western countries, a lot of new topics such as Sets, Logic and

Modern Algebra were introduced, and there were attempts to use Sets as a precise mathematical language and as a unifying concept. The effort was strongly supported by the government, and in September 1964, a prominent government school piloted the new textbooks. A new School Certificate examination syllabus was devised and was taken for the first time by students from that school in 1969. Even before the piloting was over, many schools began to jump on this modern mathematics bandwagon, and by the early seventies, more than 60% of the Hong Kong schools were following the modern syllabus.

3. 1970's

Just as more and more schools were adopting the modern syllabus, the negative effects of modern mathematics began to show up in the early seventies. Although the Extra-Mural Departments of the two universities in Hong Kong offered courses for training of mathematics teachers on modern mathematics, many teachers who were teaching the modern syllabus were still unsure of what this new curriculum was about. Some of them taught the new content in the traditional way, some supplemented modern mathematics textbooks with traditional texts that they were used to, and some gave up and reverted back to the traditional curriculum altogether. Students were unable to appreciate the conceptual and rigorous approach of modern mathematics, and they found the new topics abstract and unrelated to their everyday experience. Partly because of the neglect of pedagogical considerations in implementing the new syllabus, and partly because of the lack of good textbooks and training of teachers, the mathematics competence of students was found dropping, and science teachers complained that the mathematical knowledge of students was not adequate as a tool for their subjects (Leung, 1974).

In 1975, the Mathematics Curriculum Development Committee under the Education Department published a provisional (teaching) syllabus for the first three grades of secondary school. A similar syllabus for the fourth and fifth grades was published two years later. The syllabuses were meant to be suggested syllabuses, and for the first time, they encompassed not only subject content, but also teaching objectives, suggested teaching strategies and resources for learning. The syllabuses aimed at bringing together "the most vital elements and insights from both the 'traditional' and the 'modern' approaches" (Curriculum Development Committee Hong Kong, 1975b:1), and tried to make "a compromise between the emphasis of concepts in modern mathematics and on skills in traditional mathematics" (Brimer and Griffin, 1985:8).

By the late seventies, many new topics such as transformational geometry, elementary topology and matrices had been deleted from the HKCEE modern mathematics syllabus. At the same time, new topics had been introduced into the traditional syllabus, and the separation of mathematics into sections of Arithmetic, Algebra, Geometry and Trigonometry had been abolished. In other words, the difference between the traditional and the modern syllabuses had diminished considerably.

In 1978, the Education Department announced that from 1983 onward, the traditional and modern syllabuses would be amalgamated. The public response to the announcement was in general favourable, and some considered this as marking the end of the modern versus traditional mathematics controversy in Hong Kong (Leung, 1978). In the same year, the Education Department announced that from 1980 onwards, electronic calculators would be allowed in the HKCEE. This could have profound effect on the teaching of

mathematics in Hong Kong classrooms, and again many educators greeted the decision with favour.

In the general educational scene at large, another important event happened in 1978. The target of achieving nine years of compulsory and universal education by 1979 set in 1974 was brought forward one year, and 1978 saw the first group of all primary school leavers obtaining a place in the secondary school. The Secondary School Entrance Examination (SSEE) which many educators and parents found exerting undesirable effects on primary school children was replaced by a Secondary School Places Allocation (SSPA) scheme (see the section on education system in this chapter), and a new selection procedure was to take place after three years of secondary school to select students for subsidized senior secondary education.

So the seventies ended in optimism. The general public was pleased that their children had better educational opportunities, and in the realm of mathematics education, the confusion of the modern mathematics movement was over. Educators and the public alike were looking forward to an improved curriculum for their younger generation in the eighties.

4. 1980's

Perhaps the most important event in education in the eighties was the overall review of the Hong Kong education system by an OECD panel initiated by the Hong Kong government. The panel, chaired by Sir John Llewellyn, published its report in 1982, and

recommended a wide range of suggestions to improve the education in Hong Kong. One result of the report was the establishment of an Education Commission, which advised the governor on all aspects of education in Hong Kong.

In mathematics education, the prominent feature in the eighties was the lack of changes. Despite the fact that education was now universal up to the age of 15 and the great majority went on to senior secondary school, the mathematics curriculum remained essentially the same as when it had catered for a highly selected group. The changes in the HKCEE syllabuses were minimal, and there did not seem to be any change in the way mathematics was taught in the classroom either. One clear example of the lack of change was although calculators were allowed in public examinations, many schools still forbade their students to use calculators in their junior secondary years (Wong, 1990).

If mathematics education in the sixties was characterized by excessive changes, the hallmark of the eighties was the lack of change, and unfortunately, the decisions in both decades seemed to have been made without consideration of their effects on students.

Summary

From the descriptions above, it can be seen that in China, the mathematics curriculum has been submitted to revolutionary changes in terms of its content and orientation from time to time within the last 40 years since the founding of the People's Republic, and there is a severe dislocation of the teaching force in the country. The mathematics curriculum is

highly centralized and uniform, but recently China has reached a period when the intention is towards a greater degree of diversity, though compared with England, the curriculum is still very inflexible.

In England, the mathematics curriculum underwent drastic changes during the modern mathematics movement in the sixties and seventies. The curriculum is highly decentralized, but since the early eighties there have been moves by the government towards more centralization. Currently, the mathematics curriculum is experiencing a potentially big change because of the introduction of the National Curriculum, although it is difficult to judge at this moment the impact of this on the classroom teaching and the end results of the change.

Hong Kong has also undergone drastic changes in its mathematics curriculum in the sixties and seventies because of the influence of the changes in the British curriculum. The situation began to stabilize in the eighties, and has now reached a stage of relative stagnation. The curriculum is very centralized and uniform when compared with the curriculum in England.

In this chapter, the cultural background of the three places under study was briefly reviewed, the education system in the three places was briefly described, and the history of development of the mathematics curriculum in the three places was traced. It is against these backgrounds in the three places that the present comparative study was conducted, and the following chapters will set out the results of the comparison.

Chapter 5: The Intended Curriculum

In this chapter, the various components of the intended curriculum in the three places will be analyzed and compared based on the following documents:

China: *Mathematics Teaching Syllabus* (1982, 1986 and 1988 editions)
 published by the State Education Commission

(Unified) Mathematics Textbooks for Junior Secondary Schools
[*Algebra* books 1 to 4, *Geometry* books 1 and 2]
published by the People's Education Press, 1982-84

Teacher's Guides
[one for each of the textbooks]
published by the People's Education Press, 1984-85

Guide to Junior Secondary Mathematics Teaching
published by the State Education Commission, 1988

Hong Kong: *Syllabus for Mathematics* (Forms I-V)
 published by the Curriculum Development Committee, 1985

Examination Syllabus - Mathematics
published by the Hong Kong Examinations Authority

England: *Mathematics in the National Curriculum*
 published by DES/WO, 1989

Mathematics Non-Statutory Guidance
published by the National Curriculum Council, 1989

Mathematics for Ages 5 to 16
published by DES/WO, 1988

Mathematics Counts (Cockcroft Report)
published by HMSO, 1982

Mathematics from 5 to 16 (Curriculum Matters 3)
published by DES/HMSO, 1985

China

In China, syllabuses for schools are published by the State Education Commission (SEdC). The 1986 edition of the *Mathematics Teaching Syllabus* is a revised version of the 1982 edition, and is supposed to be a transitional syllabus before the syllabus for universal education is published (see chapter four). This edition is the syllabus that was in use when the present study was carried out. The 1988 edition is a draft version of the syllabus for universal education, but it was expected that this draft version would not differ much, if at all, from the final version (interview with Zhong, 3 December 1988).

The textbooks and the *Teachers' Guides* are published by the People's Education Press, the official publisher for educational materials, and so can be viewed as part of the intended curriculum. *Guide to Junior Secondary Mathematics Teaching*, published by the State Education Commission, is meant to guide teachers in their teaching. It amplifies what is written in the Syllabus.

Hong Kong

In Hong Kong, the only official documents on the intended mathematics curriculum are the *Syllabus for Mathematics* published by the Curriculum Development Committee under the Education Department and the examination syllabus published by the Hong Kong Examinations Authority, and the former is supposed to be a recommended teaching guide for teachers only. In effect, these two documents determine to a very great extent the

curriculum followed in the schools, through teachers' reference to the documents directly, and through the influence on the writing of textbooks indirectly (see Brimer and Griffin, 1985:8). The 1985 edition of the Syllabus was the one in use during the conduct of this study, and there were no signs of major amendments in the near future (interview with Fung, 25 September 1990).

England

The situation in England is more complicated because of the decentralized tradition of education and the Government's recent moves to more centralization. *Mathematics in the National Curriculum* is a statutory document containing the Attainment Targets and the Programmes of Study, published in March 1989. The *Mathematics Non-Statutory Guidance* is published by the National Curriculum Council alongside the statutory document, and is "designed to support the teaching of mathematics in the National Curriculum". *Mathematics for Ages 5 to 16* is the report of the mathematics working group on the National Curriculum. It also contains the joint comments of the Secretaries of State for Education and Science and for Wales on the report. *Mathematics Counts* is the report of a committee chaired by Sir W.H. Cockcroft set up in 1978 to inquire into the teaching of mathematics in schools in England and Wales (see chapter four of this thesis). Although published in 1982, it is still regarded as one of the most important documents in shaping current policies on mathematics education in the country. *Mathematics from 5 to 16* is written by the HMI after the Cockcroft report and discusses the various components of the mathematics curriculum.

Except for the statutory document, the documents are all advisory in nature or are meant to stimulate discussions on mathematics education. But since they are all published by central government agencies, the views expressed can be considered the "intended" views at the system level.

Aims and Objectives of Mathematics Education for Junior Secondary School Students

The documents in the three places all start with some affirmation on the nature of mathematics, and the aims of mathematics education are either derived from or summarized by these affirmations.

The *Mathematics Teaching Syllabus* in China (1988 edition) begins with the following statement:

Mathematics is the study of spatial forms and quantitative relations in the physical world.

It goes on to describe the usefulness of mathematics in the modern society and then spells out the aims of mathematics education for junior secondary level:

To enable students to grasp the foundation knowledge and basic skills of Algebra and Geometry, including elementary knowledge in intuitive spatial figures and statistics, further fostering of the ability of computation, development of logical thinking and spatial concepts, and ability to make use of the knowledge learned to solve simple practical problems, which are essential for every citizen in adapting to everyday life,

in participating in production and in further study. To foster in students good qualities of character and elementary Dialectical Materialistic viewpoints.

The forward to the Hong Kong *Syllabus for Mathematics* gives a list of overall objectives for the mathematics curriculum and then summarizes its view on the nature of mathematics:

(The) overall objectives are:

1. to continue the development of numeracy begun in the primary school - this includes much of the number work, and ability to cope with approximations, percentages, rates and ratios, and simple mensuration;
2. to prepare students to understand everyday applications outside the classroom
 - for example by teaching the fundamentals of statistics and probability;
3. to provide a basis for further work in Science and Mathematics
 - by teaching the use of mathematical symbolism and facility with its manipulation,
 - by developing an ability to use the basic logical patterns and conventions of reasoning,
 - by introducing necessary tools (such as the trigonometric ratios);
4. to introduce a general sense of the pattern and power of mathematics both as a tool and as part of our cultural heritage.

These objectives reflect an emphasis which treats mathematics more as a tool than a way of thought.

In England, the *Mathematics Non-Statutory Guidance* sees mathematics as providing both "a way of viewing and making sense of the world" and "the material and means for creating new imaginative worlds to explore". (paragraphs 2.1 and 2.2)

The most exhaustive list of aims is found in *Mathematics from 5 to 16*. In chapter one, 10 aims are listed and discussed in turn. The 10 aims are:

1. Mathematics as an essential element of communication
2. Mathematics as a powerful tool
3. Appreciation of relationships within mathematics
4. Awareness of the fascination of mathematics
5. Imagination, initiative and flexibility of mind in mathematics
6. Working in a systematic way
7. Working independently
8. Working cooperatively
9. In-depth study of mathematics
10. Pupils' confidence in their mathematical abilities

Discussion

The wording on the nature of mathematics in the Chinese Syllabus seems to imply a view of mathematics as a fixed body of knowledge, while the Hong Kong Syllabus clearly stresses mathematics as a tool. The English documents also stress the utilitarian nature of mathematics, but qualify this by highlighting the communicative aspect of utility. As Cockcroft (1982: paragraph 3) points out, the usefulness of mathematics arises from the fact that "mathematics provides a means of communication which is powerful, concise and unambiguous".

An analysis of the aims given in the documents from the three places shows that they can be briefly classified into two categories: utilitarian aims and intrinsic aims. Intrinsic aims are ends in themselves, while utilitarian aims are means to some other ends. Utilitarian aims can be further subdivided into those that utilize the product of mathematics and those that utilize the process of mathematics.

The following tables juxtapose the various aims in the three places:

Utilitarian Aims:

1. Utility of the Product of Mathematics:

CHINA	HONG KONG	ENGLAND
adapt to everyday life, solve simple practical problems	numeracy, understand everyday application	tackle practical problems from everyday life
participate in production		tackle practical problems from the world of work
further study	provide a basis for further work in Science and Mathematics	develop skills and understanding requires for further study and training
basis for learning the subjects Physics, Chemistry etc.		contribution to the overall school curriculum

table 5.1: Aims on the Utility of the Product of Mathematics

2. Utility of the Process of Mathematics:

CHINA	HONG KONG	ENGLAND
develop abilities in calculation, logical thinking and spatial visualization		develop the powers of logical thinking and spatial awareness; provide a concise means of communication
foster good qualities of character (correct goal of learning, strong interest in learning, strong perseverance in studying, down-to-earth scientific attitude, independent thinking, dare to innovate, and good studying habit) and elementary Dialectical Materialistic viewpoints		foster: perseverance; imagination, initiative and flexibility; Systematic working habit; self management, team working skills, and a 'can do' attitude to life's challenges

table 5.2: Aims on the Utility of the Process of Mathematics

3. Intrinsic Aims

CHINA	HONG KONG	ENGLAND
	a general sense of pattern and power of mathematics both as a tool and as part of our cultural heritage	a source of delight and wonder; appreciation of relationships within and fascination of maths; realisation of the role in the development of science and technology and of our civilisation

table 5.3: Intrinsic Aims

From table 5.1 above, it can be seen that the curricula in the three places do not differ very much in their utilitarian aims on the product of mathematics, all stressing the roles of mathematics in everyday life, in the overall school curriculum, and for further study.

A possible difference is the absence in the Hong Kong aims of mention of the world of work. There is also a subtle difference on the first point on application of mathematics to everyday life. While aims in China and England specify solving practical problems, those in Hong Kong aim at understanding the application (rather than application itself) only.

When we turn to the utility of the process of mathematics, some remarkable differences are found (see table 5.2). Firstly, statements in this category are totally absent in the Hong Kong aims. So it seems that utilizing the process of mathematics as a means for mental development and for fostering desirable characters in students is not something that the Hong Kong curriculum is aiming at. The aims of the Chinese and English curricula in this category appear rather similar, both advocating the fostering of good qualities of character. The only difference between the two seems to be the stress on ideological education (Dialectical Materialistic viewpoints) in the Chinese aims. A closer look at these aims and their official commentary, however, suggests a vital difference in what constitute "good qualities of character" in the two places. For example, "correct goal of learning" is one of the good qualities of character in the Chinese curriculum. But what is this correct goal? In the 1982 edition of the Chinese Syllabus, this "correct goal of learning" was actually specified in the aims: it is to contribute to the realization of the Four Modernizations, the overall goal of the whole nation. On the other hand, concepts such as "a 'can do' attitude to life's challenges" and "creating new imaginative worlds to explore" as set out in the English aims are alien to the Chinese. This echoes the social orientation versus individual orientation discussed in the last chapter.

On fostering Dialectical Materialistic viewpoints, the Chinese Syllabus (1986 edition) expounds what the "main elements of Dialectical Materialistic education" are (1986:2-3):

the view that mathematics originates from practice and is in turn put into practice; existence of concepts of movement and variation, mutual relation, mutual transformation in mathematics, e.g.: concrete and abstract, known and unknown, particular and general, simple and complex, numbers and forms, positive and negative, constants and variables etc.

Aims with such ideological flavour are not common in the Western educational world. Some may dismiss these Chinese aims pertaining to ideological education as mere rhetoric. This conclusion is not warranted until we have examined the implemented (and achieved) level of the Chinese curriculum. As far as intention is concerned, the Chinese believe that "the relationship between mathematics and Dialectical Materialism is very close" (Zhang, 1982), and that using the Dialectical Materialistic view to expound mathematical concepts is "beneficial to students for learning basic mathematical knowledge, and also helpful for students in forming Dialectical Materialistic viewpoints" (*Mathematics Teaching Syllabus*, 1982 edition:4).

Remarkable differences are again found in the comparison of the intrinsic aims of mathematics education (table 5.3). This time, it is in the Chinese syllabus that aims in this category are missing. It seems that for the Chinese, mathematics has utilitarian value only. It is not something worth studying for its own sake. It is important because it is a tool, a tool for the realization of the national goal of the Four Modernisations and for ideological education.

Both the Hong Kong and English aims stress the appreciation of the pattern and power of mathematics, and the realization of the role of mathematics in our culture. But the English curriculum is the only one among the three that mentions mathematics as "a source of delight and wonder". During classroom observation of mathematics lessons in the three places, it was found that students in London did seem to enjoy themselves more than their counter-parts in Beijing and Hong Kong during the lesson (but whether they were enjoying the mathematics is not clear). Chinese students in general exhibited a very serious attention to the lesson. This again echoes the finding from the literature concerning Chinese and Western differences in attitudes towards learning and studying (see chapter four).

Intended Teaching Methods

China

The Chinese Syllabus itself does not spell out the preferred methods of teaching, but merely mentions that a variety of methods should be used, and that the teaching should be "inspiring". It also stresses the importance of practice (1988 edition:6-7). The *Guide to Junior Secondary Mathematics Teaching*, on the other hand, has a section on "Teaching Methods and Innovations" (1988, Chapter 5, Section 2:63ff), and there it describes four kinds of teaching methods:

1. Exposition

This includes lecturing by the teacher, and students reading the textbook under the guidance of the teacher.

2. Discussion

This includes teacher asking questions for students to answer, and discussion between students.

3. Demonstration and Experiments

This includes teacher demonstrating using concrete objects and models, and students experimenting under the guidance of the teacher.

4. Exercise

This includes oral exercises, written exercises, doing exercises on the board, and competition between students.

Hong Kong

The Hong Kong Syllabus does not specify the intended methods of teaching, but encourages teachers "to try and experiment with their own methods and approaches as they think fit". (p.8)

The Syllabus nevertheless includes a section "Notes on Teaching" for every topic in the syllabus. Although the Syllabus says that "the given notes on teaching are NOT intended to be an exhaustive list of teaching methods" (p.8), one can go over the section and see what teaching methods are mentioned there.

The following methods are mentioned:

1. lecturing
2. demonstration using teaching aids
3. practising exercises
4. questions and answers
5. activities

England

In England, the Programmes of Study in the National Curriculum do not "dictate a particular style of teaching or learning" (para. 3.1, Non-Statutory Guidance), but the Cockcroft Report does spell out its suggested classroom practices (paragraph 243):

Mathematics teaching at all level should include opportunities for

- * exposition by the teacher;
- * discussion between teacher and pupils and between pupils themselves;
- * appropriate practical work;
- * consolidation and practice of fundamental skills and routines;
- * problem solving, including the application of mathematics to everyday situations;
- * investigational work.

Discussion

Table 5.4 below juxtaposes the intended teaching methods in the three places (those in brackets are methods implied in the documents):

<u>China</u>	<u>Hong Kong</u>	<u>England</u>
Teacher lectures	Teacher lectures	Teacher lectures
Teacher demonstrates	Teacher demonstrates	(Teacher demonstrates)
Oral exercise	(Oral exercise)	(Oral exercise)
Question/answer	Question/answer	Question/answer
		Discussion between teacher and students
Competition between students		
Discussion between students		Discussion between students
Practice/exercise	Practice/exercise	Practice/exercise
Experiments	Activities	Practical work
		Problem solving
		Investigation
Working on the board	(Working on the board)	
Reading textbook		(Reading textbook)

table 5.4: Intended Teaching Methods

From table 5.4, it can be seen that the intended teaching methods in the three places do not differ much, all suggesting a variety of teaching methods including exposition by the teacher, questions and answers, practice of mathematical skills, and practical activities. The only differences are the following three points:

1. Discussions between teacher and students are encouraged in England but are not mentioned in China and Hong Kong. One possible explanation of this difference is that in China and Hong Kong, the teacher is still considered an authoritative figure imparting knowledge to the students. So the legitimate teacher-student relation is one of teacher "teaching" the students and asking them questions to check their understanding, and never one of mutual discussion on an equal footing.
2. Competition between students is only mentioned in the Chinese documents, though the weighting given to this method is not clear. One piece of evidence on competitiveness in the Chinese classroom is that during some of the classroom visits in China, "league tables" were observed posted on the notice board at the back of the classroom showing the relative positions of the performance of students in mathematics tests. In England and Hong Kong, it seems that competition is not an intended teaching method, and in interviews with teachers in London, some of them even opined that it should be discouraged.
3. The English curriculum is the only one among the three that stresses problem solving and investigation. This is consistent with English teachers' view of mathematics as being a heuristic and flexible subject (see chapter six).

The Intended Mathematics Content

A detailed listing of the content in the three syllabuses is given in appendix G. In order to give as clear a picture of the similarities and differences between the three syllabuses as possible, some topic areas were broken down into sub-areas in classifying content instead of just following the classifications in the syllabuses (for example, Shape and Space in the National Curriculum in England is subdivided into Geometry, Coordinate Geometry and Trigonometry). Altogether, eight categories were used: Number, Measures, Algebra, Geometry, Coordinate Geometry, Trigonometry, Statistics, Probability.

The extraction of the detailed content from the Chinese and Hong Kong syllabuses was relatively easy because the content was listed clearly under different categories in those syllabuses. But the situation in England posed some difficulties. The National Curriculum gives only attainment targets and programmes of study, but not detailed listing of content other than illustrative examples. Also, the amount of time to be spent on each attainment target is not specified, unlike the practice in the other two syllabuses.

In order to extract the required information for comparison, the attainment targets for the third key stage (levels 3 to 8, see Schedule 1, Article 7, The Education (National Curriculum) (Attainment Targets and Programmes of Study in Mathematics) Order, 1989) were used as the content of the curriculum, and the number of attainment targets in each category was taken as an indication of the amount of time to be spent in that category (see DES, 1989a, Section 4.3). The researcher was aware that this did not necessarily

correspond to the actual intention, but in the absence of other indicators, this was considered to be the most rational approach.

The percentage of time spent in each category is shown in table 5.5 and a simplified chart showing the content in the three curricula is given in table 5.6 below:

		CHINA	HONG KONG	ENGLAND
Number	(N)	9.3%	18.5%	28.4%
Measures	(M)	0%	9.0%	9.5%
Algebra	(A)	46.3%	25.6%	22.1%
Geometry	(G)	35.3%	20.6%	12.0%
Co-geom.	(C)	0.9%	6.9%	1.7%
Trigo.	(T)	6.1%	7.1%	1.7%
Stat.	(S)	2.1%	10.5%	15.4%
Prob.	(P)	0%	1.7%	9.1%

table 5.5: Percentage of Time Spent on Various Mathematics Topics

[illegible]

In table 5.6, the same symbols (M, N, etc.) on the same horizontal level refer to the same topics in the countries concerned. So the chart gives an idea of the number of different topics studied in the three places as well as the commonality in content among the three places.

Discussion

A number of points can be observed from the figures in table 5.5 and the chart in table 5.6 above:

1. The Chinese put great emphasis on Algebra and Geometry. Together these two categories occupy over 80% of the total content. (In fact, the Chinese Syllabus only categorizes the content into either Algebra or Geometry: Number, Algebra, Coordinate Geometry and Statistics in our categorization all belong to Algebra, while Geometry and Trigonometry belong to Geometry).
2. Measures and Probability are not studied in Chinese junior secondary schools at all, and Statistics and Coordinate Geometry are given very little emphasis.
3. Relatively speaking, the English curriculum puts more emphasis on Number (which occupies nearly 30% of the content) and Probability and Statistics (25%), but less emphasis on Geometry (12%).
4. The topics in the Hong Kong syllabus are more uniformly distributed except for the low emphasis on Probability.

In order to inspect the relative emphases on curriculum content between the three places more clearly, the content categories above were grouped under four broader headings: Arithmetic (Number and Measures), Algebra, Geometry (Geometry, Co-ordinate Geometry

and Trigonometry), and Probability and Statistics. Then the percentage under each heading was expressed as a deviation from the mean of the percentages and bar-charts showing the deviations for the three places were plotted in figure 5.1 below:

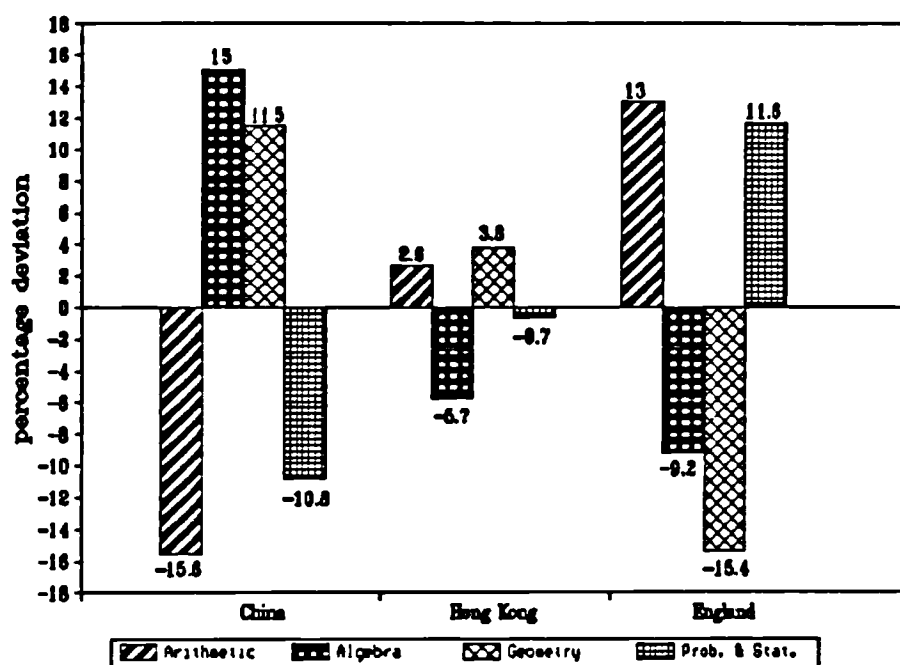


fig. 5.1: Relative Emphases on Content

From the results in tables 5.5 and 5.6 and the bar-charts in figure 5.1, it is clear that the mathematics content in the curriculum of the three places differed tremendously. For the Chinese, junior secondary school mathematics meant Algebra and Geometry. Probability and Statistics did not occupy any significant positions, and Arithmetic was a subject for the primary school only. In fact during the interviews with mathematics educators in Beijing, some of them admitted that Probability and Statistics had never gained a proper

place in the secondary curriculum in China, and some teachers even expressed doubts on whether Statistics should be considered as mathematics at all. The researcher also learned from the interviewees that all topics under the category Measures and most topics under Number were covered in the primary school and would not be revisited at secondary level.

In contrast, secondary school mathematics in England comprised predominantly Arithmetic and Probability and Statistics. These are topics that are most related to applications in everyday life, and hence it seems that the aim of applying mathematics to everyday life is better reflected in the content than other aims. Less Geometry was taught in England than in China or Hong Kong, and in particular Euclidean Geometry was given very little emphasis in the intended curriculum.

In the intended content in Hong Kong, Algebra and Geometry were given less weight than in China but more weight than in England; and Arithmetic and Probability and Statistics were accorded a higher priority than in China, but a lower priority than in England. So the intended content in Hong Kong can be considered "mid-way" between the patterns in China and England.

The emphases on Algebra and Geometry and de-emphases on Number and Probability and Statistics in China also mean that the scope of mathematics taught is narrower than that in Hong Kong and England; and the layout of content in the syllabus suggests a "linear approach" in the treatment of topics, where "a topic is treated thoroughly before moving on to the next topic, and taught topics are seldom revisited except for revision purposes" (Leung, 1987:40). During interviews with Chinese educators responsible for curriculum

development, the interviewees expressed the view that mathematics topics should either be treated thoroughly or else excluded from the syllabus. So it is not surprising that the net result is a "narrow and deep" syllabus.

The scope of mathematics taught in both Hong Kong and England, on the other hand, is broader, and the layout of content in both syllabuses suggests a "spiral approach" in the treatment of topics. Topics are treated briefly one at a time and then revisited and treated in more depth later. The resulting syllabuses are therefore "wide and shallow".

The foregoing listing of findings and discussion have presented an outline of the content emphases in the three curricula. The same topic in the three curricula, however, may mean very different things when it is actually taught in the classroom. To investigate what exactly the differences in content between the three places are, mere analysis of the syllabuses is not enough, and one has to rely on analysis of data from actual classroom observation and comparison of treatment of mathematical content in the textbooks in the three places. These will be done in chapter seven and eight of the thesis.

Chapter 6: Teachers' Attitudes towards Mathematics and Mathematics Teaching and Learning

In this chapter, the results of the questionnaire that investigated teachers' attitudes and beliefs on mathematics and mathematics education will be discussed. As mentioned in chapter three, teachers' attitudes were measured by a questionnaire supplemented by interviews with teachers. The characteristics of the teachers who responded to the questionnaire, teachers' views of mathematics, and teachers' views on mathematics education are presented below. The discussion below will be based mainly on visual inspection of the figures, but the differences in attitudes between teachers of different cities will also be tested for statistical significance using Analysis of Variance (ANOVA) tests. A probability of less than 0.01 will be taken as statistically significant.

Characteristics of the Teachers Who Responded to the Questionnaire

Results

	<u>Beijing</u>	<u>Hong Kong</u>	<u>London</u>
No. of returns	299(49.8%)	362(60.3%)	228(37.9%)
Gender percentage of male	37.4%	69.5%	48.2%
Age			
under 25	7.1%	12.7%	4.8%
25-34	15.9%	63.5%	28.9%
35-44	35.5%	19.6%	39.9%
45-54	36.8%	3.6%	17.1%
over 54	4.7%	0.6%	9.2%

Teaching Experience			
general	18.7 years	7.4 years	13.4 years
mathematics	17.4 years	6.9 years	12.6 years
Qualifications			
university graduate	29.8%	57.5%	83.3%
(mathematics major)	(28.4%)	(23.2%)	(48.2%)
tertiary institution	32.8%	42.5%	16.7%
secondary level	37.5%	0%	0%
Teacher Training			
with training	58.2%	62.4%	82.9%

table 6.1: Characteristics of Teachers Who Responded to the Questionnaire

Discussion

Statistics on background characteristics of mathematics teachers were not available in any of the three places (the statistics available in England and Hong Kong, for example, classified teachers according to their major subject of study in university rather than according to the subjects they teach in schools), and so it is not possible to compare the samples of this study with the populations they are supposed to represent. The background characteristics of the teachers from England and Hong Kong who participated in SIMS are available (Cresswell and Gubb, 1987; Brimer and Griffin, 1985), and they are remarkably similar to those obtained in this study, showing that although the sampling procedure in the present study is not as rigorous as SIMS, fairly representative samples might have been achieved.

In London, the percentages of male and female junior secondary mathematics teachers who responded to the questionnaire were nearly equal. In Hong Kong males outnumbered females, while in Beijing female teachers predominated in the sample.

The Beijing teachers were in general older (77% over 35, 42% over 45) and more experienced (mean teaching experience 18 years) than the teachers in London (40% between 35 and 44 years old and 26% over 45, mean teaching experience 13 years), while the Hong Kong teachers were youngest (76% under 35 and only 4% over 45) and least experienced (mean teaching experience 7 years).

The London teachers were in general very well qualified. Over 83% of them had a university degree (48% were mathematics specialists i.e. with a degree in mathematics), and 83% had formal teacher training. In Hong Kong, 58% of the teachers were university graduates, but only 23% had a degree in mathematics. Among those who did not have a university degree, the majority (35 out of the 43%) were graduates of the Colleges of Education (tertiary institutions that train primary and junior secondary school teachers). About 62% of the teachers in Hong Kong had formal teacher training. In comparison, the teachers in Beijing seemed the least qualified. Only 30% of them were university graduates (most of which (28%) were mathematics majors), though a large proportion of the teachers (58%) had teacher training of some sort. About 38% of them were just secondary school graduates or equivalent.

It is obvious from the above figures that the teachers in the three places differed greatly in gender, age, experience and training: with more male teachers in Hong Kong than in London and Beijing; more experienced and older teachers in Beijing; and more qualified teachers in London. In the analysis of data presented below, the ANOVA tests will employ a factorial design to take these differences into account when testing for the differences in attitudes of teachers from different cities. Four factors in addition to the

city factor will be included in the ANOVA test: gender (male and female), age (under 25, 25-34, 35-44, 45-54 and over 54), qualification (graduate specialist i.e. university graduate majoring in mathematics, graduate non-specialist, and non-graduate) and training (with and without teacher training). Any statistically significant interactions between the city factor and other factors will be noted. Interactions of three factors or more will however be ignored because no simple interpretations can be made. Again a probability of less than 0.01 will be taken as significant.

Teachers' Views of Mathematics - the Mathematics as a Process Scale (Section C, items 5 to 19)

The limitations of this instrument were discussed in volume two of the SIMS international report (Robitaille and Garden, 1989:199-200). There it was pointed out that the statements were "verbally complex", and that some wording used was ambiguous. For these reasons, the scale will be looked at here as a whole to identify trends in the answers, and the statements will not be discussed in detail one by one.

In the discussion below, the percentages of teachers who agreed or strongly agreed with the items will be used as the basis for analysis, following the practice of the SIMS report. For the present comparative study, particular attention would be given to those items on which teachers in the three places had different views.

The statements that form this scale are reproduced in table 6.2, listed in order of decreasing between-country differences. The figures in the first bracket after the statements show the percentages of the Agreed and Strongly Agreed combined for the three places Beijing, Hong Kong and London respectively, and the figure in the second bracket shows the difference between the extreme percentages. The figure in square bracket shows the mean of the three percentages, and can be used as a measure of the extent of endorsement of the statement for the three places combined.

Discussion

There is remarkable agreement between the findings of this study and the findings of the SIMS. The three items with the highest scores inside the square brackets, items 19, 10 and 11, are exactly the same three items cited in the SIMS report as the most strongly endorsed items (Kifer & Robitaille, 1989:206). These are also items where disagreement among the teachers in the three places was relatively small, as shown by the figures in the second bracket.

It is not difficult to see why item 19 is so strongly endorsed. It is a very general statement, and logical thinking has traditionally been associated with competence in mathematics. The unanimous endorsement of items 10 and 11 shows that teachers in the three places all hold relatively open and flexible views on mathematics, despite the between-city differences in these attitudes to be discussed below.

15. There is always a rule to follow in solving a mathematics problem
(87.6, 83.1, 17.0) (70.6) [62.6]
16. There has not been any new discoveries in mathematics for a long time
(52.9, 19.1, 1.8) (51.1) [24.6]
17. Mathematics is a set of rules
(57.7, 49.4, 13.1) (44.6) [40.1]
9. Mathematics helps one to think according to strict rules
(98.3, 88.1, 55.1) (43.2) [80.5]
7. There is little place for originality in solving mathematics problems
(43.1, 19.1, 6.7) (36.4) [23.0]
13. In mathematics, problems can be solved without using rules
(11.9, 35.4, 48.0) (36.1) [31.8]
14. Trial and error can often be used to solve a mathematics problem
(44.9, 55.2, 71.9) (27.0) [57.3]
18. A mathematics problem can always be solved in different ways
(82.4, 80.9, 64.7) (17.7) [76.0]
10. Estimating is an important mathematics skill
(81.4, 78.7, 95.1) (16.4) [85.1]
8. New discoveries in mathematics are constantly being made
(75.1, 59.7, 62.1) (15.4) [65.6]
6. Mathematics is a good field for creative people
(77.8, 68.2, 64.4) (13.4) [70.1]
12. Learning mathematics involves mostly memorizing
(15.8, 17.2, 4.0) (13.2) [12.3]
11. There are many different ways to solve most mathematics problems
(76.5, 82.3, 87.5) (11.0) [82.1]
5. Mathematics will change rapidly in the near future
(36.5, 28.3, 26.9) (9.6) [30.6]
19. Mathematics helps one to think logically
(98.3, 98.1, 92.9) (5.4) [96.4]

table 6.2: The Mathematics as a Process Scale

The results in table 6.2 indicate that teachers from the three places differed tremendously in their attitudes towards mathematics. The ANOVA tests show that the responses of teachers from the three places differed significantly in all except the last three items in the list in table 6.2 (i.e. items 11, 5 and 19). Although there are two items that attracted significant interactions between the city factor and other background factors (item 9: city by age, item 16: city by qualification), the overall between-city differences in attitudes towards mathematics are clear irrespective of the diverse background of the teachers.

It is interesting to find that a pattern of responses emerges across the three places. All except three items follow a trend - with the response of the Hong Kong teachers lying between those of the Beijing and London teachers. (The three exceptional items, 8, 10 and 12, are near the middle of the list, showing that they are items that attract neither strong agreement nor strong disagreement among teachers in the three places.)

There are seven items for which the percentages of Agreed and Strongly Agreed responses of the Beijing and the London teachers differ by more than 25%. The item that attracted the greatest discrepancy of responses is "There is always a rule to follow in solving a mathematics problem". 87.6% and 83.1% of the teachers in Beijing and Hong Kong respectively agreed with this statement (only 2.3% and 7.2% respectively disagreed) while only 17.0% of the London teachers agreed (64.7% of them disagreed). This statement perhaps best describes the difference in perception of mathematics between teachers in the three places. The Beijing teachers tended to view mathematics as a rule-oriented and fixed discipline, while teachers in London perceived mathematics as more heuristic and changing. And as pointed out above, the attitudes of the Hong Kong teachers lay between

these two extremes. Items 17, 13, 14 and to some extent item 7, are all related to this notion of mathematics being a set of rules to follow rather than something requiring originality or which can be dealt with through trial and error; while items 16, 9 and 7 express mathematics as a fixed and unchanging discipline.

It is not easy to account for these differences in attitudes. Further research is needed to verify any of the possible reasons suggested below. Two possible reasons can be offered here. The first is an evolutionary view which perceives people's attitudes towards mathematics as changing slowly from seeing it as a fixed discipline (or mathematics as a product) to a creative and changing one (or mathematics as a process) as the society develops and prospers. So in England, for example, people saw mathematics as a fixed discipline about a century ago, and as the country became more developed, its people began to adopt a more creative and flexible view of mathematics. According to this theory, the difference in attitudes between teachers in the three places is due to the fact that China is less "developed" than Hong Kong which in turn is less "developed" than England. Given time for these former two places to develop, their teachers will eventually adopt the same flexible view of mathematics.

Another possible explanation of these differences in attitudes focuses on the different cultures of the Chinese and the West rather than the different stages of development. The Chinese culture is known to stress compliance to orthodoxy (see discussion in the first section of chapter four). And since the introduction of Greek mathematics into the school curriculum in the late Ching Dynasty (see chapter four), this form of mathematics has been perceived as the orthodoxy, and the traditional compliance character of the Chinese

causes the teachers to stress rule-following much more than their counterparts in the West. Under this theory, the reason for Hong Kong teachers' attitudes being between those of the Chinese and English teachers is not that Hong Kong is more developed than China and less developed than England, but that culturally, Hong Kong lies somewhere between the Chinese and the Western cultures.

The different attitudes of teachers in the three places identified above should manifest themselves in the classroom teaching, and the discussion on the results of the classroom observations in the next chapter will address this issue again.

Teachers' Views on Mathematics Education

Each pertinent item in the questionnaire will be discussed in turn below. For the four items in section B, teachers were asked to rank order the answers. However, some teachers, mostly those from London, refused to rank order, and some teachers gave equal rankings to some answers. These rankings were adjusted before performing the analysis (e.g. two answers both ranked 3 will be adjusted to 3.5 each).

The Importance of Mathematics among School Subjects (Section B, item 1)

The ANOVA tests show that teachers in the three places differed significantly in their views on the importance of mathematics among school subjects. From the actual figures (table 6.3), it seems that teachers in Beijing and London valued the subject mathematics more than their counterparts in Hong Kong. More than 20% of teachers in both places thought that it was the most important school subject. In contrast, only 3% of the teachers in Hong Kong considered mathematics the most important. The first three most highly valued subjects for the three places are listed in order below (the figures are the percentages of teachers choosing that subject as most important):

<u>Beijing</u>		<u>Hong Kong</u>		<u>London</u>	
Chinese	61.5%	English	48.6%	English	70.6%
Maths	23.6%	Chinese	39.8%	Maths	22.6%
Foreign Language	14.7%	Maths	3.0%	Science	3.5%

table 6.3: Teachers' Views on the Most Important School Subjects

Understandably, the Beijing and the London teachers chose their respective languages as the most important subject, and mathematics became the second most valued school subject in both places. The Hong Kong teachers surprisingly chose English (rather than the mother tongue Chinese) as the most important subject, Chinese being pushed to second place and mathematics third.

These attitudes of the Hong Kong teachers can be understood in light of the special political situation that they are facing at the moment. Hong Kong has been a British

colony for nearly 150 years, but in 1997 its sovereignty will be returned to China. The concern of many people in Hong Kong at present is on how to maintain the territory as an international city beyond 1997, and they believe that competence in the English language is essential in this regard. It is in this context that most people in Hong Kong, including the teachers, believe that the English language is the most important school subject for their children.

Aims of Mathematics Education (Section B, item 2)

The differences in views of teachers from the three places on the aims of mathematics education are not as straight forward as the differences in attitude towards mathematics or towards the importance of mathematics among school subjects. Although the ANOVA tests still show that their views on the aims of mathematics education are significantly different, there are also significant interactions between the city and qualification factors (on response 2 "tools for other subjects" and response 5 "ability to communicate logically and concisely"), and between the city and training factors (response 5).

The following pattern of responses is based merely on the mean ratings (ratings range from 1 (most important) to 5 (least important) and the figures in brackets show the mean ratings of aims):

<u>Beijing</u>		<u>Hong Kong</u>		<u>London</u>	
tools	(1.8)	training of mind	(1.7)	communication	(2.2)
training of mind	(2.4)	communication	(2.5)	application	(2.3)
communication	(3.5)	application	(3.0)	training of mind	(3.1)
application	(3.7)	tools	(3.1)	tools	(3.2)
appreciation	(4.6)	appreciation	(4.7)	appreciation	(4.3)

table 6.4 Teachers' Views on the Aims of Mathematics Education

Teachers in all three places did not think appreciating mathematics for its own worth an important aim in mathematics education. For the four other aims supplied in the question, teachers from different places had different responses. Teachers in Beijing stressed mathematics as tools for other subjects (52% thought this was the most important aim) and teachers in Hong Kong stressed the training of the mind (52%), while teachers in London stressed the ability to communicate logically and concisely (33%) and the applications of mathematics (39%).

The views of the Beijing and London teachers were rather consistent with the aims in the respective intended curriculum. The intended aims of mathematics education in China were to contribute to the development of the country and to the development of the students' minds, and the Beijing teachers also thought that mathematics as tools and mathematics for the training of the mind were the most important aims. In England, both the intended curriculum and the teachers in London placed high emphases on the utility of mathematics as a means of communication and in everyday life. The views of the Hong Kong teachers, on the other hand, were contradictory to the curriculum intentions. The intended curriculum in Hong Kong clearly emphasized mathematics "as a tool (rather than a way of thought" (Curriculum Development Committee Hong Kong, 1985:Preface), but the Hong Kong teacher thought that training of the mind was the most important while mathematics as tools was the second least important.

The teachers' views on the aims of mathematics should have some implications on the way they teach mathematics, but in this particular study, these views seemed not to have manifested in the actual classroom teaching observed. For example, even though the

Beijing teachers thought mathematics as a tool for other subjects important, in their actual teaching, there was no incident of specifically linking mathematics to other subject areas observed. The Chinese textbooks did include some application examples to other areas, but they were not specifically stressed in the teaching observed.

The same applies to teachers in Hong Kong concerning their views on training of the mind. The main concern of the Hong Kong teachers when teaching in the classroom seemed to be the covering of the syllabus. Problem solving was not seen to have been stressed (something that would be expected from teachers stressing training of the mind), and it appeared that students were merely trained to follow rules in order to get the desired results.

With regard to the London teachers' stress on the ability to communicate logically and concisely, discussions among students were commonly found in the London classroom. However, the emphasis of the teachers in organising or allowing these discussions seemed to be the promotion of informal talk about mathematics rather than achieving "logical and concise" communication. In fact, during the classroom observations, none of the teachers were observed to be monitoring these discussions at all.

So it can be seen that there are discrepancies between teachers' professed attitudes and their classroom practice. Possibly when completing the questionnaire, teachers were responding to the questions at the level of rhetoric only, but their classroom practice exposed their true attitudes towards mathematics education. An alternative possibility is

that the questionnaire results reflected teachers' true attitudes, but the classroom constraints took precedence over teachers' attitudes in influencing classroom practice.

An exception to these discrepancies is the London teachers' stress on the applications of mathematics to everyday life. Applications of mathematics were built into many of the English textbooks, especially those used in individualised programmes such as SMILE (see chapter four), and by following these textbooks closely (see chapter seven), the aim of promoting the applications of mathematics might have been achieved for the English students.

Influences on Mathematics Content Taught in the Classroom (Section B, item 4)

Again the ANOVA tests show that the attitudes of teachers from the three places differed significantly on this item, although the city by qualification interaction terms are also significant for responses 4 (whether the content helps to train students' minds) and 5 (whether the content is useful for students in their future). Some interesting contrasts between the views of teachers in the three places appear when the average ratings of the responses and the percentages of "most important" responses are examined. When teachers were asked what should have the greatest influence on the mathematics content taught in the classroom, the following pattern of answers is obtained (the figures show the average rankings of the factors, and the bracketed figures are percentages of "most important" responses):

<u>Beijing</u>		<u>Hong Kong</u>		<u>England</u>	
syllabus	1.9(44%)	interest	2.4(34%)	interest	1.6(61%)
textbook	2.0(37%)	mind	2.6(21%)	useful	2.8(14%)
training students' mind	3.2(5%)	syllabus	2.9(29%)	mind	2.9(12%)
whether it is useful	3.8(6%)	textbook	3.5(11%)	syllabus	3.3(11%)
whether it is interesting	4.0(4%)	useful	3.7(5%)	textbook	4.4(1%)

table 6.5: Influences on Mathematics Content Taught in the Classroom

In determining the mathematics content taught in the classroom, the most important consideration for the London and Hong Kong teachers seemed to be whether the content was interesting and meaningful for students, while for the Beijing teachers, interest did not seem to be a consideration at all. This view of the Beijing teachers can be accounted for by the traditional attitude towards studying and learning of the Chinese mentioned in chapter four. For the Chinese, studying or learning is a serious endeavour and a hardship, and one is not supposed to "enjoy" the studying.

The London teachers' stress on interest and usefulness is not surprising at all in a culture that values the importance of the individual student. In interviews with the London teachers, many of them expressed the view that the main consideration in their teaching was that students should enjoy mathematics. So it is not surprising that system requirements like the syllabus and textbooks were thought to be comparatively less important.

These views of the London teachers are nearly diametrically opposed to those of the Beijing teachers. In Beijing, and to a lesser extent in Hong Kong as well, the syllabus seemed to dominate the content of mathematics taught in the classroom. The textbook was also thought to be an important determinant of the content in Beijing, and in contrast, it was not an important consideration in the minds of the London teachers at all.

Although the textbook was not thought to be an important determinant of the mathematics content taught in the classroom by both the Hong Kong and London teachers, the classroom observations revealed that in fact teachers in these two places followed the textbook rather closely in their teaching (see next chapter). In Hong Kong, the textbook was referred to from time to time during the teaching. In London, many of the lessons observed followed individualized programmes, and for these lessons the textbook or learning material was the sole determinant of the content learned by students. This is another example of discrepancies between teacher attitudes and classroom practice.

The Beijing teachers' attitudes were more consistent with their practice in the classroom. The textbook, which was written in strict accordance with the syllabus, was followed faithfully and strictly by teachers in Beijing.

The Beijing teachers' adherence to the syllabus and the textbooks may have economic reasons. In such a vast country as China where so little investment is put into education, resources are extremely scarce compared with the West. A single syllabus for all and a unified textbook are the most economical ways to achieve mass education. And in the absence of alternatives, many teachers have no choice but to follow what is provided. The scarcity of resources also gives rise to a less qualified and less competent teaching force (see background characteristics of teachers in Beijing who responded to the questionnaire), who may be less confident in making instructional decisions on their own and hence have a greater tendency to follow closely what is prescribed. But perhaps the sort of extreme uniformity in teaching styles and content taught observed in the classroom teaching in Beijing (see discussions on the preliminary visits in chapter three and

classroom observation results in chapter seven) cannot be completely explained by economical reasons. Unity and compliance to orthodoxy, as pointed out in chapter four, are traditional Chinese national characteristics, and it is more plausible that a combination of these characters and the economic constraints has brought about the extreme uniformity of teaching found in Beijing.

Factors Affecting Students' Success or Failure (Section B, item 3)

When teachers were asked what the most important factors affecting students' success or failure in mathematics were, teachers in all three places did not consider luck or family support important factors. The ANOVA tests show that the attitudes of teachers did not differ significantly for these two factors. But for the three other options, the ANOVA tests show that the differences in views of teachers from the three places are significant statistically. When the actual figures in the responses are examined, again an interesting pattern is observed (figures show mean ratings and percentages of "most important" responses):

	<u>Beijing</u>		<u>Hong Kong</u>		<u>London</u>
student's effort	1.56(61%)	effort	1.88(39%)	ability	1.73(54%)
teacher's teaching	1.98(28%)	ability	1.92(46%)	effort	2.09(27%)
student's ability	2.88(9%)	teaching	2.57(13%)	teaching	2.70(11%)

table 6.6: Factors Affecting Students' Success or Failure

It seems that teachers in Beijing believed in effort (paid by students and teachers) very much, and thought that the ability of students was not as important in determining success or failure in mathematics. This belief is related to the traditional Chinese attitude towards studying and learning discussed previously. It contrasts sharply with the attitudes of the London teachers, who in general thought that the ability of students was the most important factor. The attitudes of the Hong Kong teachers again lay between those of their Beijing and London counterparts.

The Chinese conception of ability and effort is very different from that in the West. The Chinese believe that ability is not "internal and uncontrollable", as the attribution theorists put it (see for example Weiner *et al*, 1971:96), but something that one can "develop". The Chinese proverb "Diligence can compensate for stupidity" shows that the Chinese place more importance on effort rather than ability.

Of potential significance in these Chinese attitudes towards ability and effort is that if they are manifested in teachers' classroom behaviours, they will convey to students the message that studying mathematics primarily involves hard work. This will in turn create external pressures on students to work hard as well as effect students' internalization of these values and beliefs. During the classroom observations in this study, these attitudes were not reflected in Beijing teachers' classroom teaching, but the Beijing students did seem to be more earnest towards their studies than students in Hong Kong and London. Whether these student attitudes were transmitted to them by their teachers is not clear.

Amount of Homework Expected of Students (Section A, item 5)

The stress on effort by the Chinese is reflected in the answers to the question on the amount of homework expected of students as well. The average values of the responses from teachers of the three places are shown below:

	<u>Beijing</u>	<u>Hong Kong</u>	<u>London</u>
Amount of Homework per week	218 minutes	140 minutes	65 minutes

table 6.7: Amount of Homework Expected of Students

It is obvious from these figures that they differ tremendously for the three places. The ANOVA test also confirms that the difference is significant statistically.

The Hong Kong and London results are consistent with those reported by students in Hong Kong and England in the early eighties. In the SIMS, students in Hong Kong and England reported that they spent 3 hours and 1 hour per week on mathematics homework respectively (Robitaille and Garden, 1989:79).

The figures in table 6.7 show that the Beijing students in general were expected to spend much more time on homework than their counterparts in Hong Kong, who in turn were expected to do much more homework than students in London. In order to understand the effect of this difference better, an analysis of the nature of the homework in the three places is necessary, but the sheer difference in amount of homework must mean more exposure to mathematics for the Beijing students, and is a reflection of the attitudes of the Chinese towards study, effort and practice discussed earlier.

Teachers' Attitudes Towards Gender Differences (Section C, item 1: "In general boys are better than girls in mathematics")

The ANOVA test shows that there were significant differences between the views of teachers from the three places on this statement. But the interaction effect between the city and gender (of teachers) factors is also significant. The percentages of teachers who agreed or strongly agreed with the statement and the mean ratings (1 stands for strongly agree, 2 stands for agree, 3 stands for neutral, 4 stands for disagree and 5 stands for strongly disagree) in the three places are shown in table 6.8 below:

	Agree	Mean
Beijing	34.4%	3.1
Hong Kong	77.6%	2.2
London	14.2%	3.9

table 6.8: Teachers' Attitudes Towards Gender Differences

The results show that teachers in Hong Kong tended to perceive mathematics as a male domain. More than 3/4 of them agreed with the statement. The London teachers were in contrast against this sex-bias. In the interviews, many teachers in London resented any slight suggestion that boys are better than girls in mathematics. This was perhaps especially sensitive in London because the issue of equal opportunities between the sexes was very much stressed there.

An investigation of the means table discloses the source of the significant interaction term. The means table and the corresponding graph are show below:

	Beijing	Hong Kong	London
male teachers	3.21	2.05	3.66
female teachers	3.15	2.60	4.02

table 6.9: Means Table for Item 1 of Section C

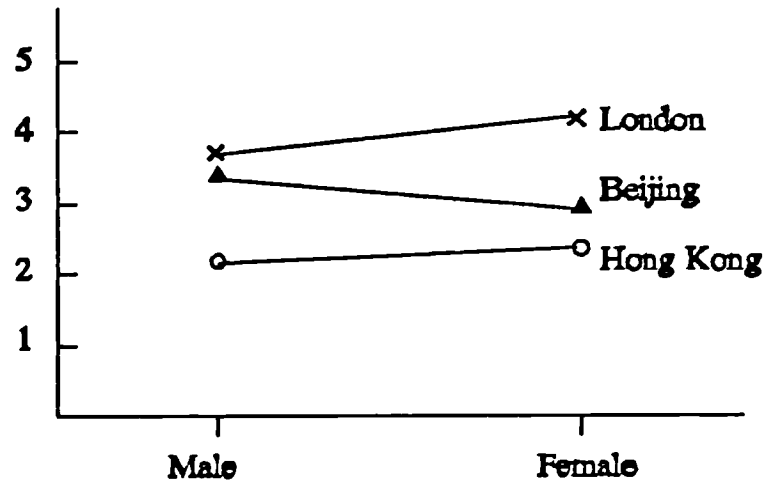


fig. 6.1: Graph of the Means Table for Item 1 of Section C

From table 6.9 and figure 6.1, it can be seen that the male teachers in Hong Kong and London were more sex-biased than female teachers, while the reverse was true for the case of Beijing.

A limitation of this item which was not found during the pilot study but was pointed out by some teachers in their returned questionnaires during the main study is the ambiguity of the word "better" used in the statement. Some teacher did not know whether "better" was referring to achievement or innate ability. From hindsight, if the adverb "innately" had been put before the word "better", the meaning of the statement would become clearer.

Section C, item 2 ("All students irrespective of their abilities can learn mathematics well")

Results:

	Agree	Mean
Beijing	17.4%	3.67
Hong Kong	26.2%	3.30
London	39.5%	3.07

table 6.10: Results for Item 2 of Section C

The ANOVA test shows that there were statistically significant differences between teachers' views on this statement, but an examination of the figures in table 6.10 shows that teachers in all three places tended to disagree slightly with the statement. Given the Beijing teachers' belief in effort rather than ability, it is a bit surprising that they did not agree with this statement more. May be they thought that not all students could learn mathematics well, but for those who could learn well their success was attributed more to their effort rather than ability. Another possible explanation hinges on the word "well" in the statement. Some teachers in Beijing said during the interviews that it was very difficult to say that students had learned "well". Perhaps the Beijing teachers were reluctant to describe their students as learning well unless they were really brilliant.

The London teachers' responses, though still slightly negative, were the most positive among the three. This again may be due to their sensitivity towards equal opportunities, and so they were relatively more reluctant to label a child as not learning well.

Some teachers remarked in their returned questionnaires that the word "well" in this item was not clear and it was difficult to say whether students had learned mathematics "well".

This perhaps has contributed to the unexpected responses mentioned above and therefore the results for this item should be interpreted with care.

Section C, item 3 ("Students in the same class but of different abilities should learn different things in mathematics")

Results

	Agree	Mean
Beijing	61.4%	2.54
Hong Kong	57.5%	2.57
London	46.4%	2.84

table 6.11: Results for item 3 of Section C

Although the ANOVA test shows that the between country F value was significant for this item, the actual figures show that the responses of teachers in all three places were rather similar (slightly more agreement with the statement), with the Beijing and Hong Kong teachers (2.54 and 2.57 respectively) agreeing with the statement more than the London teachers (2.84). It is paradoxical to find that the practices in the three places were exactly opposite to this trend of what teachers in the three places felt. Teaching in Beijing and Hong Kong was very uniform indeed, with students of different ability learning the same materials, whereas in London, individualized learning was common. Indeed, in many of the lessons observed in London, students in the same class were actually learning different things in mathematics (see the next chapter).

Section C, item 4 ("Students of roughly the same standard should be grouped together in sets for instruction")

Results

	Agree	Mean
Beijing	81.2%	2.00
Hong Kong	92.8%	1.74
London	74.2%	1.98

table 6.12: Results for Item 4 of Section C

The between city main effect in the ANOVA test is significant for this item, but so are the city by age and city by qualifications interaction terms. So it is difficult to conclude about the attitudes of teachers on this statement. The actual figures show that teachers in all three places agreed with this statement (mean ratings 2.00, 1.74, 1.98 respectively), although the school policies in the three places for setting students according to ability for instructions differed.

Grouping students in sets for instruction was not common in Beijing. In none of the schools visited for classroom observation was there any streaming of students for instruction at the junior secondary level, but between-school difference in terms of student achievement (intake of students was determined by a unified municipal examination, and the better schools were very selective) was large in general, so that students could be considered as grouped in different "sets" of schools for instruction already.

Setting students for instruction was more common in Hong Kong. In the 18 schools visited, 15 set their students for instruction in one way or another: some of them (3 schools) divided their students into two streams according to ability, some schools (3) divided students into three streams, some (6) selected one group of high ability students for instruction while the rest of the students were taught in "mixed" ability groups, and some (3) selected both a highest ability group and a lowest ability group (called remedial class) for instruction. The criteria these schools used for streaming students were usually either their achievement in English Language and Mathematics, or achievement in English Language alone, and the groups were taught in intact groups for all subjects. Only 3 schools did not set their students for instruction in any way, and they were in general schools with better student intake.

In London, there were also many schools that set their students according to ability for mathematics instruction. 9 out of the 18 schools visited set students for mathematics instruction for the whole of the secondary stage, and 5 more schools had their teaching conducted in mixed ability groups for the first two years but in sets from the third year onwards. Only 4 schools had no setting of any kind at all, most of which followed the SMILE programme.

During interviews with teachers in Beijing and Hong Kong, many of them preferred setting students for instruction because they thought that it would be easier for the teacher if students were of roughly the same ability. They did admit though that setting would create difficulty in teaching and in handling discipline in the low ability groups.

The teachers in London were more varied in their attitudes concerning setting. In many comprehensive schools where mixed ability teaching was the norm, some teachers found the job of teaching in such mode difficult to cope with, and they preferred some form of setting. Most heads of departments in those schools however favoured mixed ability teaching strongly. In the grammar schools and independent schools, teachers were more in favour of setting, and some were even strongly critical of mixed ability teaching.

In sum, from the discussions in these last two items, we can see that teachers in the three places in general thought that students of different ability should learn different things, and preferably in different sets. But the actual constraints of the school system might not allow this to happen.

Summary

In this chapter, the results of the questionnaire on teachers' attitudes towards mathematics and mathematics education have been presented. Although the ANOVA tests show that the between city differences for some of the items are not significant statistically, and that there are items with significant two way interaction effects involving the city factor, these exceptions only occur in a minority of the items in the questionnaire, and they should not distract the marked between-city differences found in the majority of the items.

So it can be concluded that teachers in the three places did differ in their views on mathematics and mathematics education, and that these differences still remained valid even when their diverse background characteristics were taken into account. More importantly, the attitudes of teachers as reflected in their responses in this questionnaire were found to follow a trend according to the cultural locations of the places they come from. Chapter nine of this thesis will try to draw on cultural and societal factors to explain these findings.

Chapter 7: The Implemented Curriculum

In the last two chapters, the intended curricula and teachers' attitudes in the three places concerned were compared based on official documents and a questionnaire. The information portrays the intentions of the systems and the teachers concerned and forms the basis for comparison of the implemented curricula, which is the focus of this chapter. Comparison for this part of the study is based on data gathered through classroom observations in the three cities of Beijing, Hong Kong and London. The arrangements for the classroom observations have already been described in chapter three. How the data were analyzed and the results of the analysis are described in the sections that follow.

Altogether 112 lessons were observed. The observations were done in 18 schools in each of the cities. The schools in these three cities differed tremendously in their physical setting, facilities and structure. The most obvious difference was the number of students in each class. The figures in table 7.1 below show the number of students in each class observed for the three cities:

	BEIJING	HONG KONG	LONDON
Mean	42.8	38.7	22.2
Maximum	54	43	30
Minimum	30	21	9
Standard Deviation	6.4	4.8	4.8

table 7.1: Number of Students in a Class

All the schools in Beijing and Hong Kong adopted whole-class instruction, with students sitting in rows facing the teacher and the teacher leading all the activities in the classroom. In London, 24 of the 40 lessons observed also followed this mode of instruction, though sometimes students sat in groups instead of sitting in rows. The remaining 16 lessons followed some individualized learning programmes, and nearly all of these lessons were conducted with students sitting in groups. These differences in class size and instructional mode in the three places should be borne in mind in the following analyses and discussions.

Method of Data Analysis

As pointed out in chapter three, classroom observations in this study were focused around a list of sensitizing concepts believed to be related to cultural differences, but attempts were also made to note down what happened in the classrooms in as detailed a manner as possible. In the analysis of results, occurrences of incidents (and if appropriate their durations as well) under the sensitizing concepts as well as those which did not fall under the concepts were taken into account. So the analysis was both "theory-driven" (attempting to interpret data drawing on cultural factors) and "data-driven" (some categories of data were generated from the data themselves). The manner this was done is described below:

First, the field notes taken during classroom observations were expanded by going over the audio-recordings of the lessons. Details missed out in the notes were filled in, and any incidents falling under the sensitizing concepts were also noted at this stage. The end results were more detailed written records of lessons than the original field notes.

Next, from these expanded field-notes, the original categories were amended so that they reflected what were captured by the classroom observations in the three places. Repeated happenings of striking events that were common to the three places or that differed greatly from place to place but which were not included in the original sensitizing concepts were noted, and new categories were constructed so that they accommodated these incidents.

At this stage, it was decided that data be treated in two different ways according to their nature: either the durations of the events were noted, or the occurrence or otherwise of the events in a lesson was noted¹. For example, the amount of teacher talk was noted as a duration while mention by the teacher of examinations would be counted as an occurrence.

Lastly, the audio-recordings were gone over again to note down the durations or count the occurrences of the incidents for analysis. In reporting the results of the analyses, the descriptions of activities in the field notes were available and the relevant parts of the audio-recordings could be traced when illustrations were needed.

¹ The alternative of counting the actual number of occurrences rather than noting whether the events occurred or not in a lesson had been considered. But since some behaviours were exhibited repeatedly by some teachers (e.g. some Beijing teachers' use of rigorous mathematical language), it would be exaggerating these occurrences of the behaviours or events if the unit of analysis was number of occurrences rather than the number of lessons (or teachers) exhibiting those behaviours or events.

The events or incidents under the two different treatments are listed below:

Events for which durations were measured:

Whole-class activities:

Symbol Event

- | | |
|----|--|
| Wo | Teacher reviews content which the class has learned previously. |
| Wn | Teacher explains new content. |
| Wb | One or more students work on the board. |
| Wq | Teacher asks questions, one or more students answer. |
| Wa | Teacher leads an activity, and students engage in essentially the same activity. |

Group Activities:

- | | |
|---|--|
| G | Students (more than one third of the class) engage in discussions or other activities in groups. |
|---|--|

Individual Activities:

- | | |
|----|--|
| Is | Individual students do the same class-work/seat-work. |
| Id | Individual students do different class-work/seat-work. |
| Ir | Individual students read (textbooks) in their seats. |
| Ia | Individual activities other than the above. |

Off-task:

- Xa** Teacher engages in administrative work while students are not working on mathematics.
- Xo** Teacher keeps discipline or teacher not in the classroom while students are not working on mathematics; confusion or other "off-task" behaviours.

Events for which occurrences were noted:

A. Events related to classroom practice

1. Elements of the teaching

- a. Teacher announces the title of the lesson or writes the title on the board.
- b. Teacher summarizes the content covered from time to time.
- c. Teacher makes extemporaneous decisions in the teaching.
- d. Teacher refers students to the textbook in the explanation or follows the textbook closely in the teaching.

2. Teacher and students activities

- a. Teacher engages in administrative work (e.g. roll-call, announcements, distribution of materials etc.).
- b. Teacher maintains discipline (e.g. reminds students to listen or to carry on with their work).

- c. A substantial number of students (more than 3) cause disruption to the teaching, much about or not engage in mathematics learning activities.

B. Events related to the intended curriculum

1. The aims of mathematics education

The teacher stresses the following in the teaching:

- a. application to everyday life
Teacher relates the mathematics to everyday life examples.
- b. tools for other subjects
Teacher applies mathematics to other (school) subject areas.
- c. training of the mind
Teacher stresses the concepts in mathematics, or points out the logic of the mathematical argument.
- d. appreciation of mathematics
Teacher points out the beauty or the power of mathematics, or teacher presents mathematics as part of our cultural heritage.
- e. mathematics as a means of communication
Teacher stresses mathematics as a means of communication, teacher encourages students to express mathematics, or more than one third of the class engage in discussion of mathematics.

2. Teaching methods

- a. Teacher stresses problem solving or employs and makes explicit problem solving strategies.
- b. (More than one third of the) Students engage in practical work or activities.

C. Events related to views of mathematics

1. Mathematical rigour

Teacher uses or requires students to use rigorous mathematical language.

2. A flexible view of mathematics

This includes:

- a. teacher or students employ alternative methods in solving the same problem,
- b. students use their own thought-out methods rather than following the teacher's method or the method suggested by the textbook,
- c. teacher or students use a trial and error method in solving problems, and,
- d. students engage in investigations or explorations.

3. Conformity and a rigid view of mathematics

These include:

- a. teacher requires students to follow the teacher's wording exactly,
- b. teacher requires students to follow fixed steps in solving problems,

- c. teacher requires students to present solutions in a fixed format, and
- d. teacher categorizes problems into standard types.

D. Events related to views of mathematics teaching and learning

1. Memorization of mathematical results and processes

This includes:

teacher requires students to memorize mathematical results or processes, or students recite mathematical results or processes.

2. Expectation on students to work hard

Teacher urges students to study hard.

3. Examinations

Teacher mentions about examinations in the teaching.

Results of the Data Analysis

The flow of events in the lessons of the three places will be described and compared based on the data on the durations of various events and the occurrences of events related to classroom practice.

Durations of the various events (see the note at the bottom of p.186):

	Beijing	Hong Kong	London (W)	London (I)
Wo mean	6.68%	9.44%	7.84%	0.00%
s.d.	9.78%	11.05%	17.01%	0.00%
i.q.r.	7.40%	15.65%	9.20%	0.00%
Wn mean	58.56%	53.06%	23.89%	0.00%
s.d.	12.49%	14.55%	15.46%	0.00%
i.q.r.	15.20%	19.70%	23.45%	0.00%
Wb mean	2.67%	4.88%	1.35%	0.00%
s.d.	4.89%	7.63%	4.98%	0.00%
i.q.r.	4.00%	8.55%	0.00%	0.00%
Wq mean	18.39%	5.03%	9.05%	0.00%
s.d.	12.91%	6.65%	12.36%	0.00%
i.q.r.	19.50%	7.80%	16.95%	0.00%
Wa mean	0.00%	0.11%	0.00%	0.00%
s.d.	0.00%	0.64%	0.00%	0.00%
i.q.r.	0.00%	0.00%	0.00%	0.00%
Sub-total for whole-class activities	86.30%	72.52%	42.13%	0.00%
G mean	0.24%	0.00%	2.97%	1.79%
s.d.	0.91%	0.00%	12.95%	4.37%
i.q.r.	0.00%	0.00%	0.00%	0.00%
Is mean	12.91%	18.93%	31.32%	0.00%
s.d.	11.14%	12.44%	22.50%	0.00%
i.q.r.	12.70%	17.15%	36.85%	0.00%
Id mean	0.00%	0.00%	5.71%	77.84%
s.d.	0.00%	0.00%	14.95%	21.16%
i.q.r.	0.00%	0.00%	0.00%	22.40%
Ir mean	0.09%	0.00%	0.00%	0.00%
s.d.	0.50%	0.00%	0.00%	0.00%
i.q.r.	0.00%	0.00%	0.00%	0.00%
Ia mean	0.21%	0.00%	1.32%	0.00%
s.d.	0.85%	0.00%	4.39%	0.00%
i.q.r.	0.00%	0.00%	0.00%	0.00%
Subtotal for individual activities	13.21%	18.93%	38.35%	77.84%

Xa mean	0.25%	1.09%	3.07%	8.31%
s.d.	0.81%	3.14%	3.62%	14.74%
i.q.r.	0.00%	0.00%	5.10%	8.50%
Xo mean	0.00%	7.46%	13.51%	12.06%
s.d.	0.00%	4.63%	9.75%	10.57%
i.q.r.	0.00%	7.30%	16.20%	22.70%
Sub-total for off-task activities	0.25%	8.55%	16.58%	20.37%

table 7.2: Durations of Various Events in the Classroom

Occurrences of events related to classroom practice:

Events			Beijing (36)	Hong Kong (36)	London (W) (24)	London(I) (16)
A.1.	a.	title	34	17	9	0
	b.	summaries	22	5	1	0
	c.	extern. decisions	0	12	8	0
	d.	textbook	32	24	2	16
2.	a.	administration	0	2	13	16
	b.	discipline	1	6	14	14
	c.	"off-task"	1	4	10	13

table 7.3: Occurrences of Events Related to Classroom Practice

Note: London (W) stands for London lessons conducted in a whole-class instruction setting, London (I) stands for London lessons following individualized programmes; s.d. stands for standard deviation; i.q.r. stands for inter-quartile range.

The flow of events during the lesson

From the data on the durations of events in the classroom, in addition to giving the mean, standard deviation and inter-quartile range of the percentages of time spent on various activities (table 7.2), it is possible to give a visual idea of the flow of events during the lesson by plotting the activities against time. "Typical profiles" for the lessons in the three cities were constructed by breaking the class time into 100 parts and using the mode for each of the parts to represent the typical activity in that part of the lesson. The typical profiles for Beijing, Hong Kong and London (whole class and individualized learning) lessons are plotted in the figures 7.1 to 7.4.

Briefly speaking, a typical Beijing lesson would start by the teacher revising with students work they had learned in previous lessons, usually done through questioning. Then the teacher would introduce the topic of the lesson (this happened in 34 out of the 36 lessons observed, and usually the topic was written on the board) and start developing the topic in some length through explanation, sometimes incorporating questioning in the explanation. One or more examples, usually taken from the textbook, would then be worked out on the board and discussed. From time to time, the content covered would be summarized (22 occasions), and students would be asked to attempt some class-work. Towards the end of the class-work time, the teacher would usually ask a few students to come out and write their work on the board. The students' work on the board would be discussed and the lesson would be summarized again. Lastly the lesson would end by teacher assigning homework.

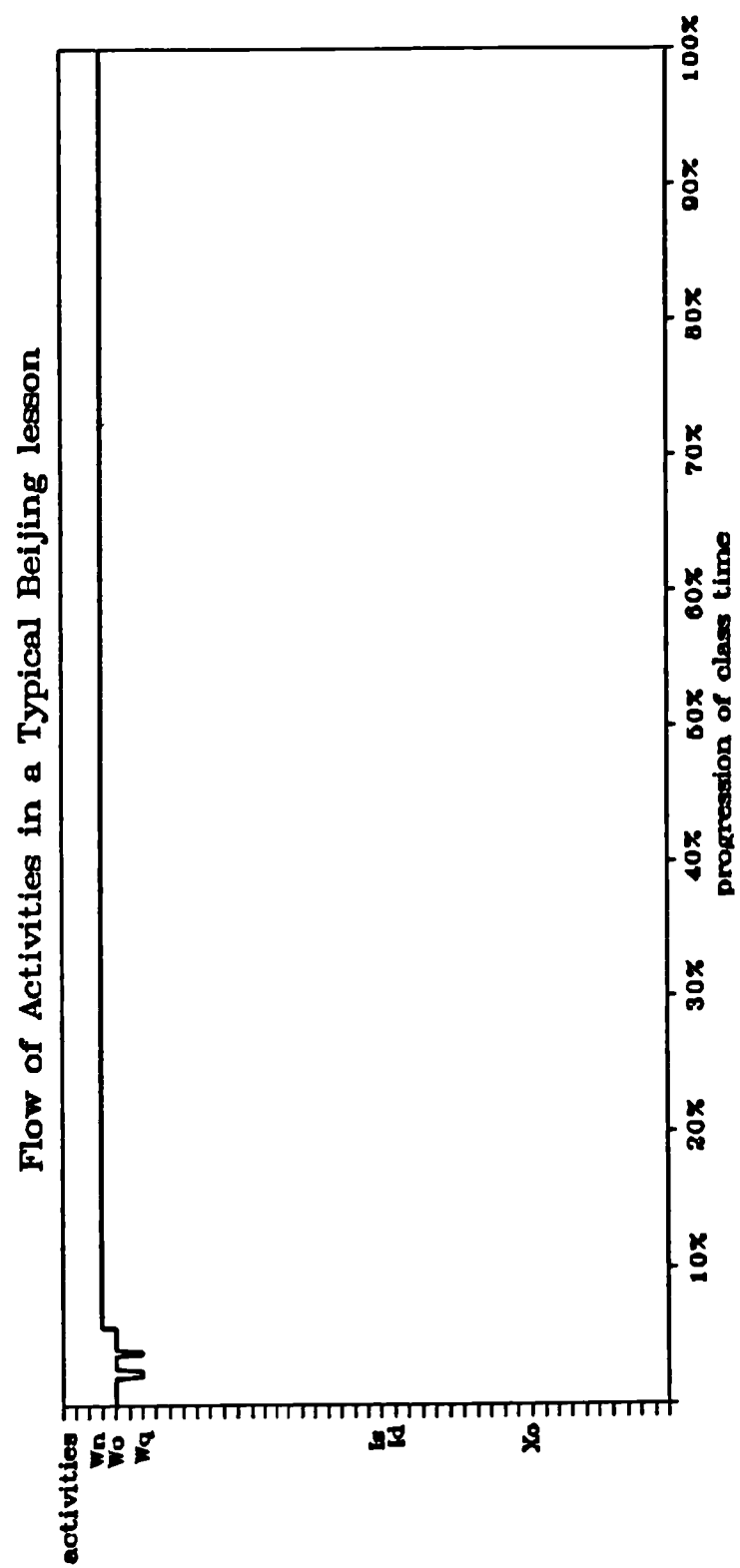


fig. 7.1: Flow of Activities in a Typical Beijing Lesson

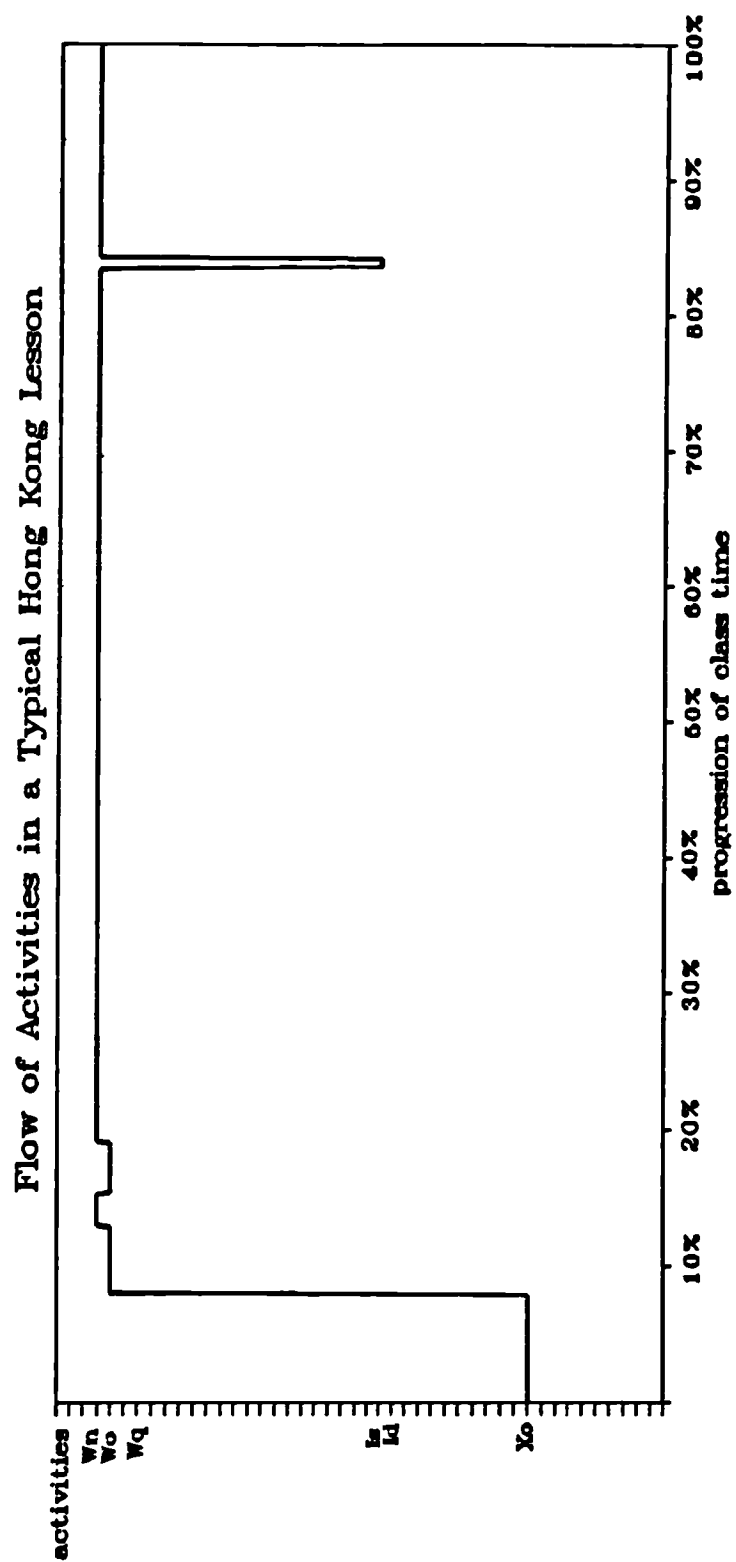


fig. 7.2: Flow of Activities in a Typical Hong Kong Lesson

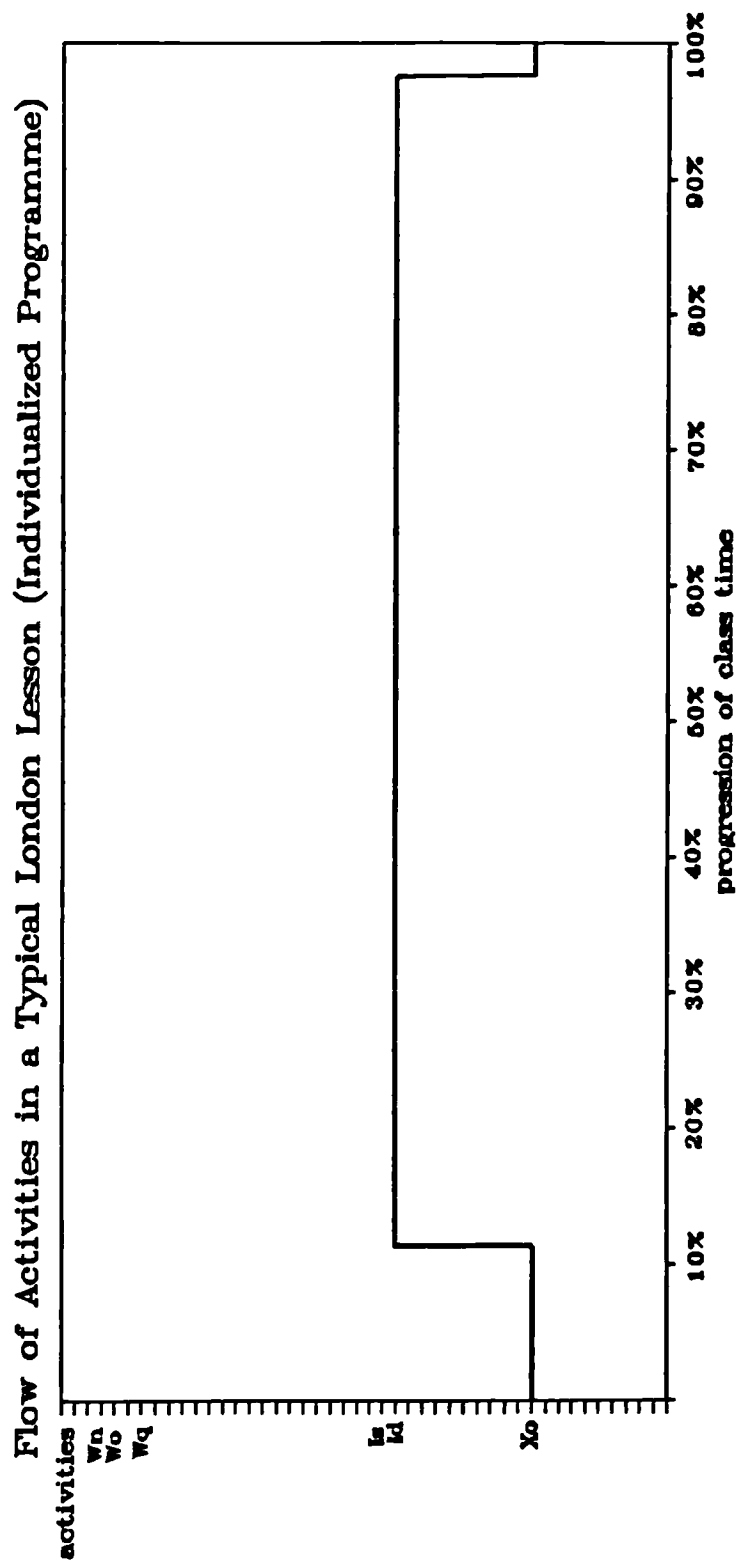


fig. 7.3: Flow of Activities in a Typical London Lesson (Individualized Programme)

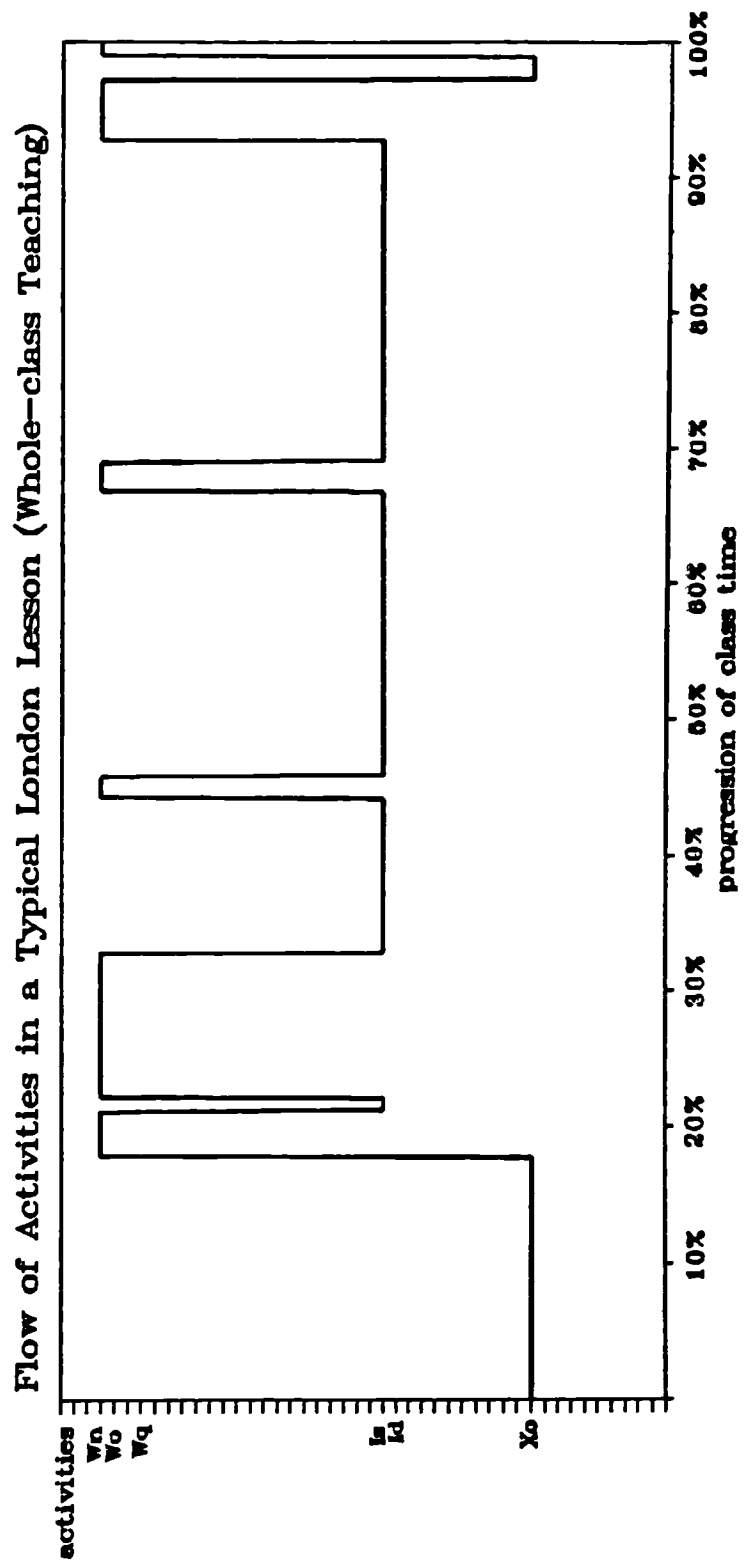


fig. 7.4: Flow of Activities in a Typical London Lesson (Whole-class Teaching)

In a typical Hong Kong lesson, the teacher would start by reminding students of what they had learned in the last lesson. Then the teacher would go on to explain new content and work examples on the board, sometimes referring students to the textbook (24 occasions). After that class-work would be assigned, and sometimes students would be asked to write their work on the board, like the practice in the Beijing classroom. Then the class-work would be discussed and the lesson would end by teacher assigning homework.

The flow of events in a London lesson that followed an individualized programme was simple. The lesson would start by teacher dealing with some administrative matters like roll-call and making sure that the students got the required materials for their work (this happened in all the 16 lessons observed). Then students would work on their own, and the teacher would go around the room busily helping students with their mathematics as well as recording the grades of students' work. At about five minutes before the bell, the teacher would ask students to "pack away", and the teacher would begin to check whether materials taken by students during the lesson were returned to the proper places. When the bell rang, the teacher would ask students to leave.

For those lessons that did not follow an individualized programme, the flow of events was very similar to that of the Hong Kong case, except that less time was spent on teacher talk and more time on class-work.

Discussion

A number of things can be noted from the above figures, the profiles of the lessons and the classroom observation field notes in the three places:

Firstly, the Beijing lessons were found to be much more structured than the lessons in Hong Kong and London, and this structure was found in nearly all the Beijing lessons observed, no matter whether the lessons took place in a key-point school in inner Beijing or a "below average" school in the remote rural areas. In most lessons, the pattern described in the paragraph above was followed strictly.

One of the reasons why the Hong Kong and London lessons appeared less structured than the Beijing lessons is that sometimes teachers in these two places were found making extemporaneous decisions in their teaching, either in the use of examples or in deciding what the next step of the lesson should be. These were found in 12 of the 36 lessons observed in Hong Kong, 8 of the 24 whole-class teaching lessons in London, but none in the Beijing classroom. In contrast, the Beijing teachers seemed thoroughly prepared for their lessons. Many of the examples used in the lessons were written on a board prepared beforehand (even though some of these examples were taken directly from the textbook), and it could be seen that each step of the lesson followed another in a structured and sometimes rigid manner.

Secondly, in spite of the intended emphasis on group work in all three places, it was rarely observed in the lessons of this study (0 to 2.97% of the lesson time). The only

frequent "group activity" observed was that students sitting next to each other occasionally discussed the class-work they were doing. This happened in all three places but was found more frequently in London¹.

Thirdly, whole-class instruction was much more common in Beijing (86.30% of the lesson time) and Hong Kong (72.52%) than in London (42.13% and 0%), whereas London students spent substantially more time doing "seat-work" (38.35% and 77.84% in contrast to 13.21% in Beijing and 18.93% in Hong Kong).

Although whole-class instruction was the predominant mode of instruction in both Beijing and Hong Kong, a closer look at the break-down of the whole-class activities time shows some major differences. The Beijing teachers used a relatively high proportion of the time (18.39%) involving students through questioning (compared with 5.03% for Hong Kong), and Hong Kong teachers used more time (9.44%, compared with 6.68% for Beijing teachers) in reviewing previous work every lesson, usually at the beginning of the lesson. As for the kind of teacher-talk in the three places, an examination of the field notes shows that there were different emphases in both the content of the talk and the manner in which the talk was conducted. Beijing teachers spent a lot of time on expounding conceptual issues, Hong Kong teachers used most of the teacher-talk time for demonstrating solutions of mathematics problems, while teachers in London spent more time discussing mathematics with students. The pace of the explanation by teachers in

¹ This was not counted as a group activity in this study unless the discussions were organized by or carried out at the instruction of the teacher. If more than one third of the class were engaged in this sort of discussions, it would be recorded as an incident of "mathematics as a means of communication". See discussion at a later section of this chapter.

Beijing was in general much slower than that in Hong Kong and London. The teacher would help students to probe into the mathematics and students were given more time to think about what the teacher was saying. In contrast, teachers in Hong Kong and London usually appeared to be rushing through the content while teaching.

In London, much more class time was devoted to seat-work. Moreover the kind of seat-work done in London was usually different from that in either Beijing or Hong Kong. For Beijing and Hong Kong, seat-work was essentially practising skills just learned from the teacher. In London, for those classes that followed an individualized programme, the seat-work (plus occasional help from the teacher) constituted the totality of the implemented curriculum for the students, and students in the same class were in general learning different things. Even for those classes that did not follow an individualized programme, there was more between-student variation in the work that they were doing compared with the lessons in Beijing and Hong Kong, because in many cases students were allowed to move on at different paces. Very often, some students were well ahead of others and of what the teacher was explaining during the whole-class instruction time, while some were lagging behind.

Fourthly, a substantial proportion of the class-time in London and Hong Kong was not used for mathematics teaching and learning, with the percentage of "off-task" time in London (16.58 to 20.37%) greater than that in Hong Kong (8.55%). In sharp contrast, in nearly none of the lessons in Beijing was off-task time observed.

The off-task time in Hong Kong was, without exception, due to the late arrival of the teacher at the classroom, and so the first 3 or 4 minutes after the bell were usually not used for mathematics teaching. But after the teacher had arrived, there were very few "off-task" times observed (4 occasions) right till the end of the lesson. In London, teachers usually arrived at the classroom late as well. But in addition, many classes would end about five minutes before the end-of-lesson bell (typically the teacher would ask students to "pack away" at about five minutes before the bell), and even during the lesson, there were occasions when the teacher was engaged in some administrative work while students were not working at all (29 out of the 40 lessons). (One example of such administrative work was that for many schools in London, the teacher had to take roll-call every lesson, whereas in Beijing and Hong Kong roll-calls were usually done before the first lesson began and so did not take up the teaching time.) These added up to give an "off-task" figure in London much higher than those in the other two places.

In Beijing, it seems that there was no off-task time during the lessons at all. As was found in the preliminary visits (see Chapter three), the Chinese teacher typically arrived at the classroom one or two minutes before the lesson, and when the bell rang, the lesson began. The lesson usually lasted right until the 45th minute (and sometimes even extending for a few minutes beyond the bell).

Fifthly, there were more incidents of maintaining discipline in London than in Hong Kong and Beijing. Incidents of teacher maintaining discipline were observed in 28 of the 40 lessons observed in London, while the corresponding figures for Hong Kong and Beijing were 6 and 1 respectively.

Lastly, it was found that the teaching in all three places depended very much on the textbooks used, although the way that the textbooks were used differed from place to place. In Hong Kong, some teachers referred students to the textbooks during their explanation of the mathematics concepts or techniques, and many teacher used the Class Practice suggested in the textbook as a learning activity for students. In Beijing, referring students to the textbook was relatively rare, but the content of most lessons followed the textbook closely. The approach of the teaching was identical to that in the textbook and the examples were in general taken directly from the textbook. In London, for those lessons that followed an individualized programme, the textbook or the individualized learning materials were the sole determinant of the students' learning. In lessons that were conducted in a whole-class setting, the influence of the textbook on the teaching was less obvious. There were only two (out of 24 lessons) incidents of reference to the textbook, and in some lessons, materials were chosen from sources other than the textbook that was supposed to be in use.

So other than the lessons that were conducted in a whole-class setting in London, all the lessons in the three places were very much influenced by the textbooks in use. In order to understand the mathematics that was learned by students in the classroom, an analysis of the textbooks would be instructive, and this will be done in the next chapter of this thesis.

Events related to the intended curriculum

Events		Beijing (36)	Hong Kong (36)	London (W) (24)	London(I) (16)
B.1.	a. application	2	1	3	0
	b. tools	0	0	0	0
	c. training of mind	0	0	0	0
	d. appreciation	1	1	0	0
	e. communication	15	1	8	6
2.	a. problem solving	9	1	0	0
	b. practical work	1	1	3	7

table 7.4: Events Related to the Intended Curriculum

Discussion

1. Aims of mathematics teaching

a. Application to everyday life

This was observed only twice in Beijing, once in Hong Kong and three times in London. Of course in the textbooks there were "application" problems, but the teachers usually did not relate these to students' everyday experience, and students seemed to tackle those problems purely as an academic exercise.

b. Tools for other subjects

This was an intended aim in all three places, but in none of the lessons observed were there any incidents of specifically applying mathematics to other subjects.

c. Training of the mind

Nothing was recorded under this category because the behaviours under this category had not been well defined before the collection of data. But it seems from the field-notes that no special attention was given to this aim in the lessons in Hong Kong and London. In Beijing, it has already been pointed out that teachers put a lot of emphasis on mathematics concepts in their explanation. Whether they did this because they thought the practice would train students minds or whether this was just a matter of "teaching habit" was not clear from the observation.

d. Appreciation of mathematics

This was an aim in the Hong Kong and English intended curriculum, but incidents under this category were only observed once in each of Beijing and Hong Kong (when the teacher related the mathematics to mathematicians in history).

e. Mathematics as a means of communication

There were many more incidents recorded under this category in the classrooms in Beijing (14 lessons) and London (15) than those in Hong Kong (1), but the kind of "communication activities" in London was very different from that in Beijing. In London, these were incidents of students sitting next to or near each other talking about mathematics. "Informal" mathematical languages were used and there was no attempt on the teacher's part to monitor the communication. In Beijing, incidents under this category happened when the teacher encouraged students in answering teacher's questions to elaborate or explain themselves more clearly (i.e. the questioning by teacher was not aimed at merely getting answers from students, but encouraging them to express themselves) to the rest of the class.

Usually, rigorous and exact mathematical language was expected (see the section on mathematical rigour below).

2. Teaching Methods

a. problem solving

This was stressed in the intended curriculum of all the three places but its occurrence was not detected in the London classrooms. It was only recorded once in the Hong Kong classroom, while in Beijing, it was recorded in 9 of the lessons. These were incidents when the teacher discussed with students the approaches to solving a problem or proving a theorem. In fact, it was a rather common practice in Beijing classrooms for the whole lesson to be devoted to discussing one or two problems in detail.

b. practical work or activities

This was recorded once in each of Beijing and Hong Kong respectively. In London, those lessons that followed an individualized programme usually included some activities that involved manipulation of concrete materials (there were 7 occasions when students were actually observed to be working with some concrete materials or apparatus), and there were 3 occasions in the whole-class lessons that students engaging in practical work were observed.

From the discussion above, it can be seen that there were discrepancies between the intended aims and teaching methods in the curriculum and the actual practice of teachers in the classroom in all three places. This point will be discussed further in chapter nine of the thesis.

Events related to views of mathematics:

Events	Beijing (36)	Hong Kong (36)	London (W) (24)	London(I) (16)
C.1. rigour	15	0	0	0
2. a. alternative methods	10	2	2	0
b. thought-out methods	0	0	0	0
c. trial and error	0	0	2	0
d. investigations	0	0	5	?
3. a. follow teacher's wording	9	0	0	0
b. follow fixed steps	14	2	0	0
c. fixed solution format	5	4	0	0
d. standard types	9	0	0	0

table 7.5: Events Related to Views of Mathematics

Discussion

1. Mathematical rigour

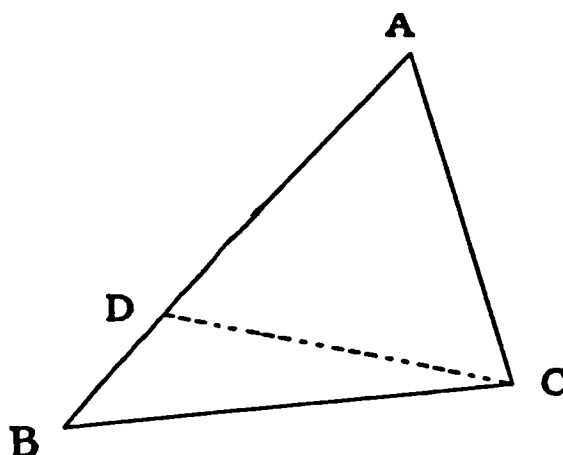
One common feature in the Beijing classrooms was the stress on the use of exact, rigorous mathematical language. This was recorded in 15 of the 36 lessons in Beijing but none in Hong Kong and London. Two striking examples are reproduced below:

Example 1 (MB0412)

It was a year 2 class and the teacher was discussing the proof of the following theorem:

"In a triangle, if two of the sides are not equal, then their respective opposite angles are not equal as well, and the angle opposite the longer side is larger."

After discussing with students how to tackle the proof and arriving at the intermediate conclusion that a line needed to be constructed in the triangle, the teacher asked the students how the line should be constructed and named a student to answer:



S: "Construct line segment CD so that AC is equal to AD."

T: "What? Which is equal to which? Are you saying this (pointing to AC) is equal to this (pointing to AD)? Where does this (pointing to AD) come from? Which do we have first? Do we have you first or do we have your father first?"

S: (silent)

(the rest of the class laughed)

T: "Can we say AC is equal to AD? How should we say it?"

S: "AD is equal to AC."

T: "Yes, AD is equal to AC."

and the lesson continued.

Example 2 (MB0111)

The lesson (year 1) was on linear inequalities in one variable, and the teacher offered a number of statement for students to judge whether they were correct or not. One of the statements discussed was the following:

T: "Judge whether this statement is correct or not. 'When the same number is added to or subtracted from both sides of an inequality, the new inequality has the same solution as the original inequality.'"

(At this point, the researcher thought the statement was correct.)

Several students tried to point out some "errors" in the statement, but without success.

The teacher continued and wrote on the board:

$$x + 5 > 2x$$

T: "Subtract $2x$ from both sides of the inequality." Teacher wrote:

$$-x + 5 < 0$$

(The students (and the researcher) began to realize the teacher's point.) The teacher then rephrased her statement:

T: "When the same number is added to or subtracted from both sides of an inequality, the new inequality with the inequality direction kept has the same solution as the original inequality."

Such is the kind of exact mathematical language that Beijing teachers were stressing.

2. A flexible view of mathematics.

a. employing alternative methods in solving the same problem

Occurrences under this theme were observed in 10 out of the 36 Beijing lessons, and only once in each of Hong Kong and London. This result is unexpected, but can be explained by the fact that mathematics content was treated more thoroughly and the pace of the lessons was slower in Beijing lessons. Typically these incidents happened when the teacher, after discussing a proof of a theorem or a solution to a problem, asked students to suggest alternative ways of proving or solving that were not included in the textbook (if the alternatives were included in the textbook and the teacher just went over them with the students, it was not counted as an occurrence of "alternative methods"). In Hong Kong and London, the explanations by teachers usually seemed to be done in a hurry, and there was in general not much time for deep thoughts or for exploration of alternative ways in tackling mathematics.

b. students using their own thought-out methods rather than following the teacher's method

The definition of what constitute students' "own thought-out" methods is difficult. There were definitely incidents when students tried to use their own methods in solving problems. Booth (1981) and Lin (1988) used the term "child methods" to describe naive strategies of solving problems adopted by children. So here the researcher was adopting a definition of "own thought-out" methods as "developed" strategies of solving problems that seemed to be thought out by the students themselves and were different from the strategies taught by the teacher.

Under this relatively stringent definition, no incident was detected under "own thought-out" methods in any of the lessons observed in the three places. May be these incidents would be more easily identified from students' written work rather than through observing classroom teaching.

c. trial and error method of solution of problems

Only 2 occurrence of encouragement of students to use trial and error methods were observed in London and none in Hong Kong and Beijing. There were two more incidents in the London lessons where students were observed to be adopting a guessing strategy in solving problems, but these were more wild guesses rather than "informed guesses" (see Polya, 1948), and so cannot be considered as trial and error. An example of trial and error is given below (ML0233):

It was a year 3 lesson and the class was doing division of numbers, and students encountered a question in the textbook which asks what numbers divide 4 will give a quotient greater than one. The teacher said to the class:

T: "I suggest you do this by trial and error. Try a number, if it is too small, try a bigger one."

An example of wild guesses is given on the next page: (ML0123)

It was a bottom set in the third year. The students were working on different things in their seat, and a student asked the teacher how many grams there were in a kilogram. The teacher drew the attention of the class and asked:

T: "Anybody know how many grams ... shuu ... can you listen ... Andrew ... how many grams in a kilogram?"

(Students all shouted answers out from their seats.)

S1: "ten."

S2: "fifty, twenty, forty."

S3: "one hundred"

S2: "thirty, thirty-five, ten, ..."

(Teacher laughed)

T: "Come on, keep going up."

S4: "fifty."

T: "More than fifty."

S4: "a hundred"

T: "More than a hundred."

S1: "two hundred."

T: "More than two hundred."

S4: "a thousand."

T: "A thousand, yes, a thousand. What does "kilo" always mean? What does the word "kilo" always tell you? ..."

And the lesson continued.

d. Investigations or Explorations

These occurred in five out of the 24 whole-class instruction lessons in London, but none in Hong Kong and Beijing. Four of the five lessons were formally investigation lessons (the teacher announced at the beginning of the lesson that the class was to do an investigation in that lesson), the remaining one was a lesson which was devoted totally to exploring patterns and making generalisations. In addition, there were some activities within individualized programmes that could be considered investigations. Here are two examples of investigations done in the London classrooms:

Example 1: (ML1723)

"There are 10 people in a room and they all shake hands with each other. How many hand-shakes are there?" (year 3)

After the students had explored the cases for 2, 3, ... 10 people, the teacher introduced the term "triangular numbers" to students. Then he asked students how many hand-shake there would be if there were 100 people, and went on to generalize to the number of hand-shakes when there were n people in the room.

Example 2: (ML1633)

"A newspaper boy has to deliver two copies of newspaper to two of five households, but he has forgotten which two. How many ways are there for the two copies to be delivered?" (year 3)

The students were then asked to explore cases for different numbers of copies of newspaper and different numbers of households, and the teacher again introduced the term "triangular numbers" to students. The teacher did not generalize the investigation further (e.g. to n copies and m households) though.

3. Conformity and a rigid view of mathematics

a. Teacher requiring students to follow the teacher's wording exactly

This was observed in 9 of the 36 lessons in Beijing lessons but not in any of the Hong Kong or London lessons. It usually occurred when students were either answering the teacher's question or working a problem on the board. The teacher, in responding to the students' answer or in going over students' work on the board, would "correct" the students' answers or work so that the wording followed that of the teacher's, even though the students' own wording was not wrong. For example: (MB1022)

It was a year 2 class and the teacher was discussing the solution of the following equation:

$$x^4 - 7x^2 - 8 = 0$$

The teacher named a student to offer a solution and wrote it on the board:

Solution: Let $x^2 = y$. Then original equation becomes

$$y^2 - 7y - 8 = 0$$

$$(y - 8)(y + 1) = 0$$

$$y = 8, y = -1$$

The student continued:

S: "Because x square is equal to y , therefore x square is equal to 8 or x square is equal to ..."

T: "You better say 'When y is equal to 8, x square is equal to 8'."

Teacher wrote her own version on the board:

$$\text{When } y = 8, x^2 = 8 \quad x = 2\sqrt{2}$$

$$\text{When } y = -1, x^2 = -1 \quad \text{no real solution}$$

...

b. teacher requiring students to follow fixed steps in solving problems

This was a more common practice in Beijing (14 occurrences) than in Hong Kong (2 occurrences), but was not found at all in the London classroom. It was meant to help students solve mathematics problems systematically. For example, in a year one lesson on applications of linear equations in Beijing, the teacher started the lesson in the following way: (MB0211)

T: "What are the steps for solving application [word] problems on [linear] equations?"

(pause) "Mary."

Mary recited: "The steps in solving application problems are: one, examine the question; two, set up the unknowns; three, list the equivalent quantities; four, express using algebraic expressions; five, set up the equation; six, solve the equation."

T: (to the whole class) "Is that correct?"

The whole class: "Yes."

Incidentally, this is also a good example of encouraging students to memorize mathematics processes.

c. teacher requiring students to present solutions in a fixed format

Related to the last theme were incidents when the teacher dictated a format of solution to mathematics problems for students to follow. This happened five times in Beijing and four times in Hong Kong. What happened was the teacher would write down a worked example on the board or discuss a worked example in the textbook thoroughly and then require students to follow the format when they solve that type of problem. It was meant to help students present mathematics systematically, but the rigid manner this was imposed on students might result in encouragement of conformity in expressing mathematics.

d. teacher categorizing problems into standard types

This was a practice found only in some Beijing lessons (9 occurrences) and not in any of the Hong Kong and London classrooms. For example, some Beijing teachers would classify problems of applications of linear equations into a number of types (mixing liquids, two people or cars travelling at different speeds on the same road, a number of people doing a piece of work together or separately etc.), and would deal with each type of applications one by one.

From the discussions above it can be seen clearly that the classroom practice of Beijing teachers reflected a more rigid view of mathematics than the practices in Hong Kong and London. This is consistent with the finding on teacher attitudes in the last chapter, and this point will be elaborated further in chapter nine.

Events related to views of mathematics teaching and learning

Events	Beijing (36)	Hong Kong (36)	London (W) (24)	London(I) (16)
D.1. Memorization	16	2	0	0
2. Expectation	0	0	0	0
3. Examinations	5	4	6	1

table 7.6: Events Related to Views of Mathematics Teaching and Learning

1. Memorization of mathematical results and processes

Incidents of encouragement of memorization were observed in 16 lessons in Beijing, two lessons in Hong Kong and none in London. These were incidents when the teacher asked the students specifically to memorize some mathematical facts, or when the teacher asked students for the statement of a theorem or some mathematical results and students were expected to recite the results. An example from a year two class in each of Beijing and Hong Kong is given below:

Beijing (MB1322)

T: "What is the theorem for the properties of isosceles triangles and its two corollaries? ... Susan."

Susan: (recite by heart) "The theorem for the properties of isosceles triangles: 'The two base angles of an isosceles triangle are equal.' Abbreviation, 'Equal angles opposite equal sides.' Corollary one: 'The angle bisector of the apex angle of an isosceles triangle bisects and is perpendicular to the base,' Corollary two: 'All the angles in an equilateral triangle are equal, and each angle is equal to sixty degrees.'"

T: "Can you repeat the second corollary again?"

Susan: "All the angles in an equilateral triangle are equal, and each is equal to sixty degrees."

T: "Sit down." (to the whole class) "The way she put it was not too bad."

[This is supposed to be a compliment to Susan for her answer.]

The teacher was introducing the topic of simple and compound interest, and he explained the terms "interest", "principals", "amount" etc. to students. Then the teacher said to the students:

T: "Now copy these terms onto your notebooks. You have to memorize these terms and their meanings."

Then students copied what were on the board onto their notebooks. After that, the teacher went over the terms again with students.

2. Expectation on students to work hard.

No incident at all under this theme was observed. May be expectation on students to work hard was a view not held by teachers in all three places, or may be the expectation was conveyed to students in more subtle ways not detected by the observations in this study.

3. The influence of examinations

Mention by the teacher of examinations was recorded in five lessons in Beijing, four lessons in Hong Kong and seven lessons in London. An example from a year three lesson in London is given below: (ML0723)

The lesson was on Pythagoras Theorem, and the students were working on some problems on the theorem in their seats while the teacher was going around helping students with their work. Then the teacher drew the attention of the students:

T: "Everyone draws a triangle out. Get used to drawing triangles out because you often can't decide what to do until you saw a drawing of it. And even if you can, you get marks for showing drawings. The examiner doesn't know what you are thinking ..."

Mentioning examinations is of course only one manifestation of the influence of examinations. Whether the content covered in the lessons is confined to those found in the examination syllabus and whether the kind of exercise done in class is devised so that it is similar to that encountered in examinations are other manifestations. It was mentioned above that classes in all three places followed the textbooks very closely. As textbooks in China and Hong Kong were very much influenced by the examination syllabus when they were written and included mainly paper and pencil type of exercise while those in England (especially those for individualized programmes) seemed to be less influenced by the examination syllabus and contained more activities, it could be argued that Beijing and Hong Kong lessons were more influenced by public examinations if we look at students' activities rather than listening to the teacher's words during the lessons.

The discussions above show that there was a much greater stress on memorization in the teaching in Beijing. This is consistent with the finding in the previous section that the teaching in Beijing reflected a more rigid view of mathematics held by teachers. On the other hand, the finding concerning examinations points to a common phenomenon in all three places: the classroom teaching in all three places was influenced to some extent by examinations.

Summary

In summary, the findings from the classroom observation show that there was much more concern for individual differences in London, as manifested by schools adopting individualized learning programmes, and by teachers spending less time on whole-class instruction and more time on seat-work and allowing students to proceed at different paces. In Beijing and Hong Kong, students were more exposed to direct instructions from the teacher. In Beijing, mathematics lessons were more structured and there was greater stress on mathematics concepts, while in Hong Kong more emphasis was put on practising mathematical skills. In London, the stress was on students doing mathematics on their own, and for those lessons that followed an individualized programme, students were interacting with the textbooks or work-cards most of the time. Also, students in Beijing were being asked more questions by the teacher and had more opportunities to express mathematics verbally in front of the whole class. They also spent less time during lessons on off-task activities. To put it crudely, it can be said that Beijing teachers emphasized the content or concepts of mathematics, Hong Kong teachers emphasized mathematical skills, while London teachers emphasized experiencing and enjoying mathematics.

Other findings from the classroom observation data are that the teaching in Beijing reflected a more rigid view of mathematics held by teachers than those in the other two places, that the teaching in all three places was influenced by examinations, and that there were great discrepancies between the intended curriculum and classroom practice in the three places. This last finding will be elaborated in chapter nine of the thesis.

The results presented above also show that the methodology adopted in this study for classroom observation and for analysis of the observation data does help to identify differences in classroom practices that are related to cultural differences. But the results also indicate that the cultural explanation does not account for all the differences, and other factors (such as practical constraints in the classroom) have to be drawn upon for an adequate explanation. The relative strengths of different kinds of factors in explaining different classroom practices will also be discussed in chapter nine.

Chapter 8: Content of the Implemented Curriculum: Analysis of Selected Topics from the Textbooks in the Three Places

Introduction

It was pointed out in the last chapter that one finding from the classroom observations was the great influence of textbooks on the implemented curriculum in all three places. Thus, an analysis of the textbooks used in the three places will enable us to understand the similarities and differences in mathematics content of the implemented curricula between the three places. To conduct a detailed analysis of the textbooks used in the three places would be beyond the scope of this thesis. What is attempted below is an analysis of the treatment of selected topics from the most popular textbooks from each of the three places.

The policies of textbook usage differed in the three places. In China, other than some experimental textbooks used on a trial basis (usually in key-point schools), the whole of the Chinese student population used the same set of textbooks published by the People's Education Press, the government publisher for educational materials¹. (There will be some changes in this policy in the near future, see Chapter four.) For the junior secondary level, the textbooks were published in two series: Algebra (1 to 4) and

¹ All the 18 Beijing schools visited in this study used this series of textbooks, but in one of the key-point schools visited, one of the class observed used an "experimental" textbook.

Geometry (1 and 2). Algebra 1 and 2 were used in the first year of secondary school, Algebra 3 and Geometry 1 in the second year , and Algebra 4 and Geometry 2 in the third. A teacher's guide accompanied each book and gave suggestions on teaching. Some local districts also published additional exercise booklets to supplement the exercise found in the textbooks.

In Hong Kong, schools were free to choose textbooks published by commercial publishers, but all textbooks had to be scrutinized by a committee under the Education Department. When this study was conducted, there were about 17 sets of textbooks approved by the textbook committee to be used in schools. The most popular series was "Mathematics for Hong Kong" published by Canotta Publishing Company (1988), which claimed about 70% of the market (information supplied by the publisher)². The series was published in five volumes, one for each of the five years of secondary school. There were "Workbooks" and "Supplementary Guides" available to accompany the textbooks, and "Solution Guides" which contained "suggested step-by-step solutions to all questions in the textbook" (p.vi, Preface) were available to teachers who used the series.

In England, schools also chose their own textbooks. In addition, in some schools, different grades used different textbooks and some grades used a combination of different textbooks. The most popular set of textbooks was the SMP 11-16, which was used in about 50% of the schools in U.K. (information supplied by the publisher, Cambridge

² Of the 18 Hong Kong schools visited in this study, 12 of them used this series of textbooks.

University Press)³. For the first two years of secondary school, the text was organised into booklets at four successive levels, with each level divided into parts (a) and (b). For levels 2 to 4, there were extension booklets (e) for the more able. The booklets were also accompanied by worksheet masters, review booklets, teacher's guides, answers books and learning aids. The booklets were designed for individual work, but there was "scope for other teaching methods" (p.10, information booklet of the series) as well.

In years 3 to 5, the textbooks were designed for students grouped in sets for teacher-led instructions, and there were four series of textbooks catered for different levels of attainment. The textbooks were also accompanied by worksheet masters, teacher's guides, supplementary booklets and extension books. A "Green series" was published in eight volumes (G1 to G8) for low attainers (below the 60th percentile) and prepared students for assessment at 16+ by a graduated assessment scheme with certificates awarded by the Oxford and Cambridge Schools Examination Board. There were also booklets for students to work through individually in year 3, and supplementary booklets to enable students to obtain a GCSE grade. The "blue series" and the "red series" shared the same first two volumes (B1 and B2), and differentiated into two series for three more volumes each (B3, B4, B5 and R1, R2, R3). They were for students in the "broad middle band", from the 25th to the 60th percentile. The "yellow series" was also published in five volumes (Y1 to Y5) and was meant for the highest attainers (top 20-25 percent). There were two extension books to this series (YE1, YE2) for the most able students.

³ Of the 18 London schools visited in this study, 6 of the schools used the SMP series, and 3 other schools used SMP plus some other textbooks.

The comparison below was based on the People's Education Press' Algebra 1 to 4 and Geometry 1 and 2 in China, and volumes 1 to 3 of the Canotta textbooks in Hong Kong. In England, the choice of which books from the SMP series for analysis was a more difficult decision because there was no clear-cut practice of which volumes were to be used in a particular year. It depended very much on how fast the class or the individuals were proceeding. In the comparison below, the English materials were chiefly based on the level 1 to 4 booklets, the first three volumes of the Green series, and the first two volumes of the Blue and the Yellow series. But the other volumes would also be briefly referred to when needed.

Four topics were chosen for comparison, one from each of the areas of Numbers, Algebra, Geometry, and Statistics. For each topic in each of the three sets of textbooks, the content will be briefly described and some examples from the textbooks will be given. Then the treatment in the three places will be compared and discussed.

Negative Numbers

China

The topic of "Negative Numbers" was treated under chapter one "Rational Numbers" of Algebra 1 and was therefore studied in year one. It was the very first topic in secondary mathematics, and so the pre-requisite knowledge assumed was only the arithmetic (four rules of positive integers and fractions) learned in primary school.

Negative numbers and related topics were treated in the following order and manner:

1. The idea of negative numbers was introduced as "quantities with opposite meaning" using the example of temperatures below zero degree. (pp.1-2)
2. Integers, Fractions and Rational Numbers were defined. (p.4)
3. The representation of rational numbers using a number line was discussed. (pp.5-6)
4. The idea of (additive) inverse was introduced. (pp.6-7)
5. The absolute value of numbers was defined ("The absolute value of a positive number is the number itself; the absolute value of a negative number is its (additive) inverse; the absolute value of zero is zero."). (pp.8-9)
6. Comparison of the magnitudes of rational numbers using the number line was discussed ("For two numbers represented on the number line, the number at the right is greater than the number at the left."). (pp.10-11)
7. Rules for addition of rational numbers were discussed. (pp.16-18)

("1. When two numbers of the same sign are added, take the same sign and add the absolute values of the two numbers.

2. When two numbers of different signs are added, take the sign of the number with the greater absolute value, and subtract the smaller absolute value from the greater absolute value. The sum of opposite numbers (additive inverses) is zero.

3. A number added to zero is still the number itself.")
8. Laws governing addition of rational numbers (commutativity and associativity) were discussed. (pp.19-21)
9. Rules for subtraction of rational numbers were discussed ("To subtract a number is the same as to add its additive inverse."). (pp.25-26)

10. The laws governing addition of rational numbers were extended to subtraction. (p.27)
11. Rules for multiplication of rational numbers were discussed. (pp.32-34)

("When two numbers are multiplied together, (the product is) positive if their signs are the same, negative if their signs are different, and we multiply their absolute values together.

Any number multiplied to zero gives zero.")
12. Laws governing multiplication of rational numbers (commutativity, associativity and distributivity) were discussed. (pp.36-38)
13. Rules for division of rational numbers were discussed. (pp.39-40)

("When a number is divided by another, (the quotient is) positive if their signs are the same, negative if their signs are different, and we divide the absolute value of one by that of the other.

Zero divided by any non-zero number gives zero.

Note: Zero cannot be the divisor.")
14. Powers of rational numbers were discussed ("Powers of positive numbers are positive; odd powers of negative numbers are negative, even powers of negative numbers are positive."). (pp.46-48)
15. Rules for mixed problems were discussed ("First calculate the powers, then calculate multiplications and divisions, and lastly calculate additions and subtractions. If there are brackets, calculate those (operations) inside the brackets first."). (p.49)

A lot of exercise was provided for practice. 75 items were given in the text as oral or written exercise for practice in class, and 283 more items were given in the Exercises after

the text, giving a total of 358 items. 24 of them were application problems. Some examples of these items are reproduced below:

Exercise 1 (pp.12-16)

27. The yields of four experimental varieties of wheats, type 1, type 2, type 3, and type 4, compared with that of type A are as follows (yields greater than that of type A are positive):

Type 1: +12.4%; Type 2: -9.8%;

Type 3: -6.4%; Type 4: +8.6%.

Which of the four experimental varieties has the highest yield? Which has the lowest yield?

Exercise 3 (pp.28-32)

17. Calculate:

(4) $|(+3)-(-4)|$, $|+3|-|-4|$.

Exercise 5 (pp.51-53)

6. (1) Are 3^2 and 3×2 the same? Why?

(2) Are 3^2 and 2^3 the same? Why?

13. Calculate

(12) $(0.01 - 0.03)^3 - (2 \times 0.04^2 - 0.0015)$.

Hong Kong

The topic "Negative Numbers" was treated under chapter 9 "Directed Numbers" of Book 1, and so was studied in year one. There were three relevant topics before chapter 9 in Book 1: "Fundamental Arithmetic" (chapter 1) revised the four rules for whole numbers and fractions learned in primary school, and also played the role of introducing students to arithmetic terminology in English; "Numbers and Counting" (chapter 2) introduced the concepts of numbers and numerals, and went on to discuss the binary system; and "Introduction to Algebra" (chapter 3) introduced the use of letters to represent numbers, and discussed simple equations in one unknown.

In chapter 9, the notions of directed numbers and negative numbers were also introduced using the example of temperatures above and below zero degree, like the treatment in the Chinese textbook. The meaning of Integers was introduced in a footnote. The representation of directed numbers on a number line was then discussed. Addition of directed numbers was presented with the aid of the number line, adding a positive number meant moving right on the line and adding a negative number meant moving left. Subtraction of directed numbers was also discussed using the number line, and was introduced as the reverse of addition. Multiplication of a negative number by a positive number was introduced as repeated addition of that negative number, and the product of two negative numbers was induced from a pattern. Division of directed numbers was treated as the reverse of multiplication.

54 exercise items appeared in the Class Practice for use during lessons, and 121 more items appeared in the Exercises that followed the text, giving a total of 175 items. Only 5 of the items related to some physical situations. Some examples are given below:

Exercise 9A (pp.225-226)

8. Simplify the following expressions.

(e) $(9 - 1.8) - (-8.2 - 11) - 20$

(f) $-\frac{5}{8} - (\frac{1}{8} - \frac{1}{4}) + (\frac{5}{8} - \frac{3}{4})$

Exercise 9B (pp.231-232)

11. $(1 - 0.2) \times 1.5 + 3.6 + 3$

21. $-2\frac{1}{8} + \frac{-3}{-4} - \frac{1}{-2}$

England

Topics related with negative numbers were found in booklets 2(a) (Negative Numbers) and 3(e) (Patterns with Negative Numbers), and books G3, B1, Y1 and R1. So students were usually first introduced the idea of negative numbers in year 1 (booklet 2(a)). For the "low attainers", the topic would probably be encountered again in year 3 (book G3). For students in the "broad middle band", the topic would be encountered again one or two more times, roughly in year 3 (B1) or year 4 (R1). For the "highest attainers", the topic

would probably be encountered two more times, in years 2 and 3 (3(e) and Y1) respectively. Relevant topics in junior secondary school before the first encounter of negative numbers in booklet 2(a) included a thorough study of whole numbers (6 booklets), fractions (2 booklets) and decimals (2 booklets).

In booklet 2(a), the idea of negative numbers was introduced and amplified using the example of temperatures below zero degree. Then the adding of positive numbers to, and subtraction of positive numbers from, negative numbers were discussed using the notions of rise and fall in temperature.

For "low attainers", the topic in book G3 (p.28) was merely a review of the ideas learned in booklet 2(a). No new content was introduced. For students in the "broad middle band", chapter 11 in book B1 (pp.83-93), in addition to revision of ideas learned previously, introduced addition and subtraction of negative numbers using a concrete example (the skating contest). For the "highest attainers", addition and subtraction of negative numbers were introduced in booklet 3(e), and multiplication (introduced using the idea of enlargement with a scale factor) and division (introduced using the notion of division as the reverse process of multiplication) in chapter 5 of book Y1 (pp.57-63). In each of books B1 and Y1, a short section was devoted to "negative numbers on a calculator".

For both the booklets and the textbooks, there was relatively little exposition or discussion in the form of texts, and the content consisted mainly of exercise for students to work on. The average number of exercise items for each stream of students was about 360. About

a quarter of the items related to some physical situations. Some of these items are reproduced below:

Booklet 2(a) (pp.14-16)

D7 The record drop in temperature in 24 hours was at Browning, Montana, USA. On 23rd-24th January 1916, the temperature fell from 7 C to -49 C.

(a) How many degrees did it fall?

(b) Write a subtraction beginning 7 - ...

E4 Here is an addition.

$$-3 + 5 = 2$$

Here is a 'story' for this addition.

A newt starts 3 metres below water level.

It climbs 5 metres up the bank.

It is now 2 metres above water level.

Make up a story for this addition.

$$-7 + 4 = -3$$

Your story does not have to be about a newt!

Book Y1 (p.62)

B7 Work out

(l) $-4 - -9$

(m) -4×-9

(n) $-30 + -6$

(o) $(-4)^2$

Discussion

The first thing to be noticed is that whereas the Chinese and Hong Kong textbooks assumed that all students were to learn the same thing concerning negative numbers, different students were expected to learn up to different levels in the English textbook. The manner in which the idea of negative numbers was introduced in the three places was similar. All started with the use of negative numbers in denoting temperatures under zero degree, but the treatments immediately after the introduction of the idea differed from place to place.

After the introduction of the notion of negative numbers, both the Chinese and Hong Kong textbooks went on to discuss the representation of negative numbers on a number line, while the English text still dwelled on the example of temperatures and developed further ideas from the example. The denoting of temperatures on a scale is in essence representation on a number line, but it was not presented as such, and further ideas developed were bound to the physical situation of temperatures (rise and fall of temperatures etc.). So it seems that while the Chinese and Hong Kong texts used the example of temperatures below zero degree merely as motivation or set induction to bring out more abstract concepts, the English text treated the practical situation itself as content to be studied.

After the discussion on representation of negative numbers on a number line, the Chinese text presented the ideas of (additive) inverse and absolute value, and used them as tools

to introduce rules concerning addition and subtraction of negative numbers. The text then went on to generalize laws (commutativity and associativity) governing the addition of rational numbers. The Hong Kong text, on the other hand, went on directly to discuss addition and subtraction of negative numbers with the help of the number line.

Since the English text was still dwelling on the example of temperatures, only addition of positive numbers to and subtracting positive numbers from negative numbers were discussed. For the "low attainers", they were not expected to learn beyond this eventually, and for others, they were introduced to addition and subtraction of negative numbers in a later book or booklet. There addition and subtraction of negative numbers were introduced by observing patterns (see Appendix H), and the ideas were consolidated using concrete situations. Then calculations involving negative numbers using calculators were discussed, and students in the "broad middle band" were expected to learn no further than this during junior secondary school.

For multiplication and division of negative numbers, the Chinese text used a concrete example (rise and fall of water level with time) to discuss the multiplication of a negative number by a positive number. From that discussion, a rule was generalized and applied to multiplication of two negative numbers, and the text went on to discuss laws governing multiplication of rational numbers. Division was then introduced as the reverse of multiplication.

In the Hong Kong textbook, multiplication of a negative number by a positive number was presented as repeated addition of that negative number, and the rule for the

multiplication of two negative numbers was generalized from a pattern. Division was introduced as the inverse of multiplication.

In the English textbook (book Y1, i.e. for high attainers only), multiplication was presented as enlargement by a factor, and the idea was extended to multiplication of negative numbers. The approach was in essence observation of patterns, but pictorial patterns instead of number patterns were used. Division was also presented as the reverse of multiplication, but the discussion was done in a very non-technical language.

To illustrate the differences in approach, the treatments of subtraction of negative numbers in the three places are reproduced in Appendix H. It can be seen from appendix H that the treatments in the Chinese text and the Hong Kong text were similar, both introducing subtraction of negative numbers as the reverse of addition. The difference between them was that while the Hong Kong text referred the argument to a relatively more concrete model of the number line, the Chinese text relied solely on pure argument, and the treatment was therefore more abstract. The treatment in the English text was more concrete and comprehensive, and a variety of approaches were used.

From the discussions above, it can be seen that the treatment in the Chinese text was much more conceptual. Instead of stopping at calculations involving negative numbers, laws (commutativity and associativity) governing the operations of negative numbers were introduced. The Chinese text also generated a lot of "rules", and from the classroom observation results discussed in the last chapter, it can be seen that many of these rules

were actually memorized by students. The Hong Kong text also generated rules, but these were rules to aid calculation rather than rules stated rigorously and precisely as in the case of the Chinese text. The English text also gave rules to aid calculation, but the emphasis was clearly on recognition of patterns, and the rules were meant to be generalizations of the patterns observed.

Quadratic Equations and Quadratic Functions

China

The topic Quadratic Equations was found in chapter 11 of Algebra 3; and chapter 14 of Algebra 4 contained topics on Quadratic Inequalities and Graphs of Quadratic Functions. So Quadratic Equations and Quadratic Functions were studied in year 2 and year 3 in China.

In Algebra 3, quadratic equations were introduced through an example and then the general quadratic equation ($ax^2 + bx + c = 0$) was defined. The text went on to discuss four methods of solution of quadratic equations: taking square root, completing square, by formula and factor method. The first two and the fourth methods were introduced using examples, and the quadratic formula was derived by solving $ax^2 + bx + c = 0$ through completing square.

The text then introduced the discriminant of the general quadratic equation $ax^2+bx+c=0$, and application examples of quadratic equations were given. Next the sum and product of roots (in terms of the coefficients of the quadratic equation) were discussed, and the text went on to give examples on construction of quadratic equations given the roots and factorization of quadratic trinomials using the quadratic formula.

The next section in the chapter tackled the solution of equations reducible to quadratic equations. These included equations of degree higher than 2, algebraic fractional equations and irrational equations involving square roots of algebraic expressions. The last section of the chapter dealt with simple systems of quadratic equations.

In Algebra 4, the topic Graphs of Quadratic Equations was introduced through two concrete everyday life examples, and terms related with the quadratic graph (parabola, axis of symmetry and vertex) were given. Then graphs of equations of the form $y=ax^2$ were discussed using the graphs of $y=x^2$, $y=-x^2$, $y=\frac{1}{2}x^2$, and $y=2x^2$, and the properties of the graphs of $y=ax^2$ were generalized. Next the graphs of $y=(x+3)^2$ and $y=(x+3)^2-3$ were discussed in terms of translations of the graph of $y=x^2$, and the text went on to discuss the graph of $y=ax^2+bx+c$ by first transforming it to the form $y=a(x+\frac{b}{2a})^2+\frac{4ac-b^2}{4a}$. The coordinates of the vertex $(-\frac{b}{2a}, \frac{4ac-b^2}{4a})$, the equation of the axis of symmetry $(x=-\frac{b}{2a})$ and the maximum or minimum value of y $(\frac{4ac-b^2}{4a})$ were then expressed in their general form. The section ended by listing the properties of the function $y=ax^2+bx+c$.

In a later section of the same chapter, quadratic inequalities were discussed using the quadratic graph as an aid. The section started by defining what is meant by a quadratic inequality, and then used the graph of $y=x^2-x-6$ to discuss the inequalities $x^2-x-6>0$ and $x^2-x-6<0$. The results were generalized to the solution of $ax^2+bx+c>0$ and $ax^2+bx+c<0$ with the aid of the graph of $y=ax^2+bx+c$, and the cases when the discriminant was greater than, smaller than and equal to zero were discussed. The summary in the chapter summarized what were discussed above in the form of a table.

There were 202 items given in the text as oral or written exercise for practice in class, and another 187 items were found in the Exercises after the text, giving a total of 389 items for practice. 30 of the items were word problems, 26 of which related to applications in everyday life. Some examples of these items are given below:

Algebra 3

Exercise 8 (pp.133-136)

7. Use the method of change of variables to solve the following equations:

$$(2) \ x^2+3-\sqrt{2x^2-3x+2}=\frac{3}{2}(x+1)$$

9. Solve the following equations in x :

$$(1) \ \frac{2x}{x-a}=\frac{12x^2}{a^2-x^2}=\frac{a-x}{x+a} \ (a \neq 0)$$

Exercise 9 (pp.148-149)

6. Solve the following systems of equations:

$$(4) \begin{cases} x^2 - 3xy + 2y^2 + 4x + 3y = 1, \\ 2x^2 - 6xy + y^2 + 8x + 2y = 3. \end{cases}$$

8. If the width of a rectangle is increased by 1 cm and its length decreased by 1 cm, then its area will be increased by 3 cm². Given that the original area of the rectangle is 12 cm², find its length and width.

Algebra 4

Exercise 7 (pp.80-81)

4. Given that the parabola $y = ax^2 + bx + c$ passes through the three points A(0,1), B(1,3), C(-1,1), determine the values of a, b, c and draw the parabola.

Exercise 8 (pp.93-94)

7. Prove that for any real number k, the equation

$$x^2 - (k + 1)x + k = 0$$

has real root(s).

Hong Kong

The topic Quadratic Equations was found in chapter 8 of book 3 and was therefore studied in the third year. The chapter started by introducing quadratic polynomials through an

example, and went on to discuss factorization of quadratic polynomials. Then quadratic equations were introduced using an example, and after pointing out the fact that $ab=0 \Rightarrow a=0$ or $b=0$, the text went on to illustrate solving quadratic equations using the factor method through a number of examples. Next the text introduced quadratic graphs and went on to graphical solution of quadratic equations. The chapter ended with worked examples on "easy" (word) problems leading to quadratic equations.

Topics on Quadratic Equation, Quadratic Functions and Quadratic Inequalities would be revisited again in Book 4 and 5 (i.e. in senior secondary school), where more thorough treatment would be provided.

In Book 1 to 3, there were altogether 5 exercise items given in the text as Class Practice, and 222 items were given in the Exercises at the end of the text. 20 of the items were word problems, but only 3 of them related to some application in everyday life. Some examples are given below:

Exercise 8D (pp.220-221)

Find the roots of the quadratic equations:

39. $-11 - 14x = -16x^2$

In each of the following, form a quadratic equation in x with the given roots:

68. $\frac{3}{2}, -\frac{1}{3}$

Exercise 8E (pp.227-228)

9.(a) Draw the graph of the equation $y = 4x^2 - 12x + 9$ from $x = 0$ to $x = 3$.

(b) Hence solve the quadratic equation $4x^2 - 12x + 9 = 0$.

(c) Does the equation $4x^2 - 12x + 9 = 0$ have a double root?

Exercise 8F (pp.230-231)

17. A rectangle has a diagonal 10 cm long. If the length of the rectangle is 2 cm longer than its width, find the length and width of the rectangle.

England

The topics Quadratic Equations and Quadratic Functions were not found in the textbooks at junior secondary level except in booklet 4(e) (U-Shaped Graphs) where one of the U-shaped graphs discussed was that of a quadratic function. But there students were only expected to plot the graph of a quadratic function derived from an everyday life example, and to find the maximum value of the function from the graph. The term quadratic was not even mentioned.

In senior secondary school years, topics involving quadratic functions and equations were found in Books R3, Y4 and Y5, but detailed treatment of the topics was only found in Book Y4. So higher ability students would typically learn something on these topics near the end of their secondary school years. There the treatment of the topics was more graphical than algebraic, and solution of quadratic equations by graphical method (R3 and Y4), factorization (Y4) and decimal search (Y4) were discussed.

Discussion

It can be seen from above that the depth of treatment of the topics differed greatly from place to place. At junior secondary level, the topics were given much greater weight in China than the corresponding treatment in Hong Kong, while junior secondary school students in England were not expected to learn the topics at all. Even for English students in senior secondary years, only the high ability students had the chance of exposure to these topics. And what Chinese students learned in their junior secondary school years on these topics were even more advanced than what Hong Kong and English students learned in their senior secondary school years.

The treatment of the topics in China was abstract and thorough. There was much manipulation of symbols involved, and a lot of topics related to quadratic equations and quadratic functions were covered. In the Hong Kong textbooks, the emphasis at the junior secondary stage seemed to be on introducing this idea of quadratic equations. The topics were not meant to be treated in detail and difficult problems were excluded at this stage. When the treatment in the English textbooks (at the senior secondary level) was compared with that in Books 4 and 5 (i.e. for senior secondary school) of the Hong Kong series and the Chinese textbooks (at junior secondary level), it was found that the approach in the English text was more concrete, relying heavily on graphs. The Chinese text was more conceptual in approach, and the approach in the Hong Kong senior secondary textbooks was rather similar to but less conceptual than that in China.

Isosceles Triangles

China

The topic "Isosceles Triangles" was found in chapter 3 (Triangles) of Geometry 1 and was therefore studied in the second year after the introduction of basic concepts in Geometry, Intersecting and Parallel Lines, and Congruent Triangles. This part of the syllabus was "a systematic study of (this) special kind of triangles on the basis of (the knowledge of) congruent triangles" (Teacher's Guide, 1984:72).

The definition of isosceles triangles ("Triangles with two of the three sides equal are called isosceles triangles.") and the various terms associated with isosceles triangles (legs, base, vertical angle and base angles) were given in a previous section on classification of triangles according to the relation between the three sides (p.83). The section on Isosceles Triangles contained two subsections. The first subsection started by asking students to perform an "experiment": to fold an isosceles triangle so that the two equal sides coincide and "discover" that the two base angles are equal. Then the text proceeded to prove formally 'the theorem for the property of isosceles triangles': "The two base angles of an isosceles triangles are equal." Two corollaries of the theorem were then stated and three worked examples were given, one of which was a proof of a theorem ("In a triangle, if two of the sides are not equal, then their respective opposite angles are not equal as well, and the angle opposite the longer side is larger."). The second subsection gave a

formal proof of 'the theorem for the determination of an isosceles triangle': "If a triangle has two equal angles, then their corresponding opposite sides are also equal." Again two corollaries of the theorem were stated and three worked examples were given (one of which was the proof of the theorem: "In a triangle, if two of the angles are not equal, then their respective opposite sides are not equal as well, and the side opposite the larger angle is longer.").

The proof of the theorem in the second subsection is reproduced below (p.114):

Theorem for the determination of isosceles triangles: "If a triangle has two equal angles, then their corresponding opposite sides are also equal." (Abbreviation: equal sides face equal angles)

Given: In $\triangle ABC$, $\angle B = \angle C$ (figure 3-35).

To prove: $AB = AC$.

Proof: Construct the angle bisector AD of $\angle BAC$.

In $\triangle BAD$ and $\triangle CAD$,

$$\begin{cases} \angle 1 = \angle 2 & \text{(definition of angle bisector),} \\ \angle B = \angle C & \text{(given),} \\ AD = AD & \text{(common side),} \end{cases}$$

$\therefore \triangle BAD \cong \triangle CAD$ (AAS).

$\therefore AB = AC$ (corresponding sides of congruent triangles equal).

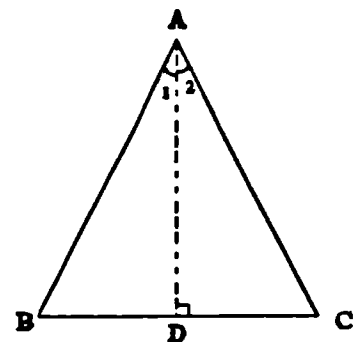
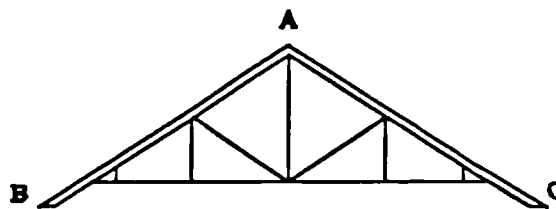


Fig 3 -35

There were 15 items in the Exercise that followed the text proper. All of them were word problems (problems expressed in words) and diagrams were given for 9 of the items. Two of the items were "application problems", applying the properties of isosceles triangles to some physical situations. Two examples of the items on this topic are given below:

Exercise 8 (pp.117-119)

1. Given that the lengths of the beams AB and AC of a house are equal and their included angle is 118° . Calculate the angle between a beam and the horizon BC.



15. In $\triangle ABC$, $AB=AC$, D is a point on BC produced, E is point on AB, DE and AC intersect at point F. Prove that: $AE < AF$.

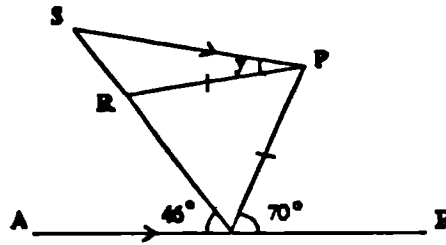
Hong Kong

The topic "Isosceles Triangles" was found in Book 3 under chapter 4: "More About Congruence and Similarity", and was therefore studied in the third year. The definition of an isosceles triangle was first given in Book 1 (year one) under the subsection "4.4 Triangles" of chapter 4: "Fundamental Geometry" when the classification of triangles according to the measure of their sides was discussed. By the time students encountered this topic in year 3, they should have learned the topics Basic Concepts in Geometry, Angles and Parallels, and Congruent Triangles. The section on Isosceles Triangles in Book 3 started by introducing the definition of an isosceles triangle ("If two sides of a triangle are equal in length, the triangle is isosceles.") and the various terms involving an isosceles triangle (vertex, vertical angle and base). Then through a step-by-step class activity, the property of the equality of the base angles of isosceles triangles and its converse were introduced. The formal proofs of the property and its converse, and three worked examples were then given.

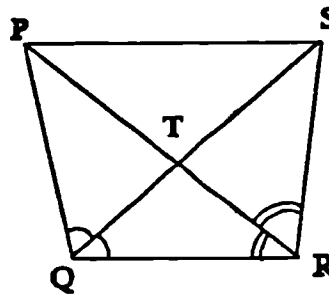
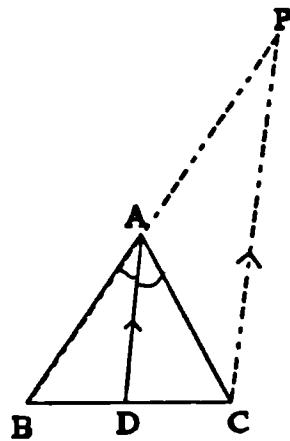
The exercise in the text contained 26 items, 10 of which were word problems with diagrams given. None of the items related to a physical situation. Examples of the items are given below:

Exercise 4B (pp.82-85)

12. (In the figure, find the value of the unknown.)



24. The angle bisector of $\angle BAC$ in $\triangle ABC$ cuts BC at D . Through C a line is drawn parallel to DA and this line meets BA produced at P . Prove that $AP=AC$.
- *26. In a quadrilateral $PQRS$, QS and PR are the angle bisectors of $\angle PQR$ and $\angle SRQ$ respectively. If $TQ = TR$ and $PST = 45^\circ$, prove that
- $\triangle PQT \cong \triangle SRT$,
 - PR is perpendicular to QS .



note: (*) indicates harder questions

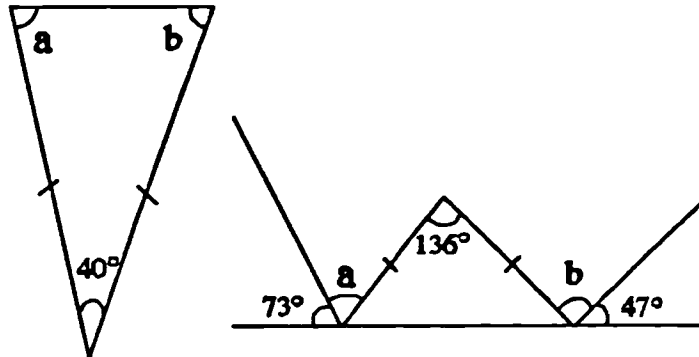
England

The topic Isosceles Triangles appeared in Booklet 2(e) (Angle Relationships) and Book B2, and so would be studied by high attainers in year 1 and by students in the "broad middle band" in year 2. When students first encountered this topic, they should have learned some basic concepts in geometry including angles and parallels (but not congruent triangles). The treatment in both the booklet 2(e) and Book B2 was very brief, consisting of only a few lines of text, and no worked examples were given.

In the booklet, the section under isosceles triangles only consisted of the definition of an isosceles triangle ("An isosceles triangle is one with a pair of equal sides."), and the statements "The angles at the corners B and C (referring to a diagram in the text) are equal. They are often called the 'base angles'.". In Book B2, the relevant text also only consisted of three statements: "An isosceles triangle is one which has two equal sides. An isosceles triangle has a line of reflection symmetry. Two of the angles of an isosceles triangle are equal.".

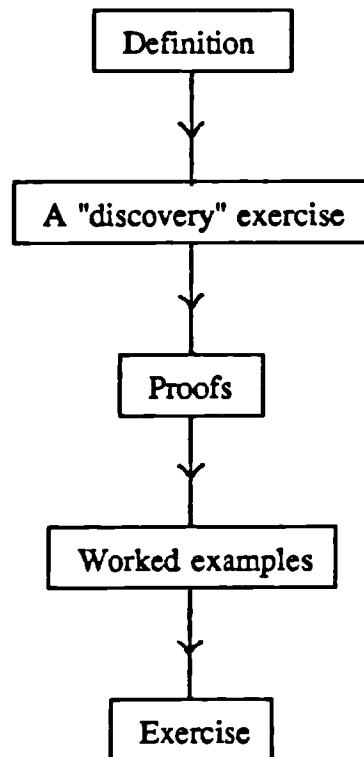
The exercise in the booklet consisted of 9 items and that of Book B2 7. All the items, without exception, asked students to find unknown angles marked in given diagrams. There were no word problems or problems that required proofs. Some examples of these items are given below:

Find the angles marked a and b.



Discussion

The texts on this topic in China and Hong Kong were amazingly similar, following the following pattern:



The treatment in the English text was very brief, consisting merely of a statement of facts and applying the facts to some given triangles. This shows that this topic was given much less weight in England than in China and Hong Kong.

The number of items in the exercises in China, Hong Kong and England was respectively 15, 26 and 8 (average), showing that the Hong Kong textbook provided more exercise for practice than the Chinese and English ones. The fact that no word problems appeared in

the English text meant that students were not required to interpret mathematical problems from words for this topic. They were merely required to solve highly idealized problems presented in diagrams. For Hong Kong, diagrams were always given in the word problems, and so although students were exposed to problems expressed in words, they were not required to translate the words into mathematical models (diagrams). The translation was done by the author of the textbook and the mathematical models were provided. In China, all the items in the exercise were word problems, so students were expected to interpret mathematics problems expressed in words, and for those problems with no diagram given, students had to construct the diagrams (mathematical models) themselves. The Chinese text also attempted to link mathematics with everyday life applications, although the emphasis was not too strong (2 items out of the 15).

The greatest difference in treatment of this topic in the three places is the emphasis on proofs. There was a much greater emphasis on geometric proofs in China than in Hong Kong, while geometric proofs (for this topic at least) were not expected of students in England at all. Five of the six worked examples in the Chinese textbook were proofs, while one of the three worked examples in Hong Kong was a proof. In the exercises, 12 of the 15 items in the Chinese textbook required proofs, 7 of the 26 items in the Hong Kong textbook required proofs, while none of the items in the English text required proving. This difference in emphasis on proofs perhaps reflects a deeper difference in the understanding of what Geometry or even what Mathematics is.

Statistics

China

The topic Statistics was found in the last chapter (chapter 16: Introduction to Statistics) of Algebra 4, and so was studied by students at the end of their third year.

The topic was introduced through two concrete examples and then the terms population, element, sample and sample size were defined. Then the idea of mean was introduced using an example and generalized to the formula:

$$\bar{x} = \frac{1}{n} (x_1 + x_2 + \dots + x_n) , \quad \text{and rewritten in the form of}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Calculation of the mean using an assumed mean was then discussed, and the idea of weighted mean introduced.

Next the idea of variance was introduced using an example and generalized to the formula:

$$s^2 = \frac{1}{n} [(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2] \quad \text{and rewritten as}$$

$$s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

Then the idea of standard deviation was introduced, and formulas for simplification of the calculation of the variance were discussed.

The next section in the text dealt with frequency distribution tables and histograms for presentation of grouped data, and the last section briefly discussed cumulative frequency tables and cumulative frequency polygons.

The text contained 23 items for practice in class, and 17 more items appeared in the Exercises at the end of the text. Some examples of the items are given below:

Exercise 12 (pp.176-177)

1. The heights of 20 members of the football team in a secondary schools are as follows (unit: cm):

170	167	171	168	160	172	168	162	172	169
164	174	169	165	175	170	165	167	170	172

Calculate the mean height (to the nearest cm).

6. If the means of two sets of numbers x_1, x_2, \dots, x_n and y_1, y_2, \dots, y_n are \bar{x} and \bar{y} respectively, what is the mean of the new set of numbers

$x_1+y_1, x_2+y_2, \dots, x_n+y_n$? Why?

Exercise 13 (pp.191-192)

1. Two men A and B shoot at a target 10 times under the same conditions, and the scores they get are as follows:

A	7	8	6	8	6	5	9	10	7	4
B	9	5	7	8	7	6	8	6	7	7

Find the variances of the two samples above. From the results, who has a more stable performance?

Exercise 14 (pp.202-204)

1. 60 pieces of cotton fibre are sampled from a bulk and their lengths are measured.

The results are as follows (unit: mm):

82	202	352	321	25	293	293	86	28	206
323	355	357	33	325	113	233	294	50	296
115	236	357	326	52	301	140	328	238	358
58	255	143	360	340	302	370	343	260	303
59	146	60	263	170	305	380	346	61	305
175	348	264	383	62	306	195	350	265	385

List the sample in a frequency distribution table, and draw a histogram showing the distribution.

Hong Kong

Topics on Statistics were found in chapter 15 (Introduction to Statistics) of Book 1, chapter 14 (Frequency Distribution and Its Graphical Representation) of Book 2, and chapters 13 (Measures of Central Tendency) and 14 (Uses and Abuses of Statistics) of Book 3. So Hong Kong students would study Statistics every year in junior secondary school. The topics and their sequence of presentation are given below:

After a brief introduction of what is meant by Statistics and a brief discussion on the collection and organization of data, chapter 15 of Book 1 discussed the various ways of presenting data (Pictograms, Bar Charts, Pie Charts and Broken Line Graphs). Then grouping data into frequency tables and presenting data using histograms were introduced. Chapter 14 of Book 2 carried on with what had been covered in Book 1 and introduced Frequency Polygons and Curves. The text then went on to discuss Cumulative Frequency Polygons and Curves.

Chapter 13 of Book 3 introduced Mean, Median and Mode as three measures of central tendency. Using an assumed mean to calculate the mean was then illustrated using an example, and the text went on to discuss determination of the mean from a frequency distribution for the cases of ungrouped and grouped data respectively. A discussion on median and determination of the median from histograms and from cumulative frequency polygons followed, and then mode and modal class were discussed. The chapter ended by making a brief comparison of the three 'averages'. The last chapter (chapter 14) of

Book 3 discussed the uses and abuses of statistics using a number of everyday life examples.

There were 47 exercise items in the text for practice in class, and the Exercises at the end of the text contained a further 254 items for practice. Some examples of these items are reproduced below:

Book 1

Exercise 15C (pp.379-380)

Construct a frequency distribution table and draw a histogram for each of the following distributions:

4. The average time spent on watching television every day in a class of 40 students.

Time (min)	1-30	31-60	61-90	91-120	121-150	151-180	181-210
Frequency	1	5	13	6	5	6	4

Book 2

Exercise 13J (pp.383-384)

In Nos. 7-9,

(a) find the mean and median of the following sets of numbers;

(b) determine which one is a more representative figure.

7. 12, 18, 9, 99, 27, 3, 9, 27, 6

Book 3

Further Practice 14 (pp.417-421)

8. The percentage increase in sales of three different products in a firm are given as follows:

Product	X	Y	Z
Sales in 1986	\$40 000	\$60 000	\$50 000
Sales in 1987	\$58 000	\$76 800	\$60 000
Percentage Increase	45%	28%	20%

Comment on each of the following:

- (a) Staff A claimed that the sales had been increased by 93%.
- (b) Staff B argued that the correct percentage should be 31%.
- (c) The manager said that they were both wrong.

England

Topics on Statistics were found in Booklets 2(b) (Frequency Tables and Graphs), 3(e) (Mean and Spread), 4(a) (Grouping), and 4(b) (Comparing), and Books G2 and Y2, and so were studied in all the first three years for the majority of the students. (G6, G8, B3, B5, R1, R2, R3, Y3 and Y4 in the 4th and 5th years also contained topics on Statistics.)

Booklet 2(b) started with an everyday life example, organised the data into a frequency table and presented them in the form of a "stick graph" (bar chart). The booklet ended with some suggested activities for recording results. Booklet 4(a) discussed representation of data using "frequency charts" (histograms). Booklet 4(b) started with an example in which data were presented in a pie chart, and the text went on to discuss comparison using the mean.

Booklet 3(e) used a concrete example to introduce the range and the mean of a set of data for the purpose of comparison. The focus of Chapter 7 (Percentages) in Book G2 was on the meaning and representation of, and calculations on, percentages, and representation of data using piecharts was discussed. Chapter 8 (Distributions) in Book Y2 discussed presenting distributions using frequency charts (histograms), and the chapter ended by pointing out some limitations or abuses of the mean of a set of data.

There were 60 exercise items in Booklet 2(b) (Frequency Tables and Graphs), 56 items in Booklet 3(e) (Mean and Spread), 31 items in Booklet 4(a) (Grouping), and 55 items in Booklet 4(b) (Comparing). There were 34 items on Statistics (representation of data) in chapter 7 of Book G2, and 50 items in Chapter 8 of Y2. So students were exposed to 180 to 252 (average 216) exercise items in Statistics during their junior secondary school years. Some examples of these items are given below:

Here are some football results from a newspaper.

League Division 1			
Bolton	1	C. Palace	1
Brighton	2	Norwich	4
Bristol C.	0	Arsenal	1
Everton	0	Man. Utd.	0
Ipswich	1	Middlesbro'	0
Man. C.	0	Liverpool	4
Southampton	1	Leeds	2
Stoke	3	Derby	2
Tottenham H.	1	Nottm. F.	0
W. Brom. A.	4	Coventry	1
Wolves	1	Aston Villa	1

- B1 Prepare a tally table like this.

Result		Frequency
Home win		
Away win		
Draw		

Total

Run your finger down the football results.
 Make a mark in the tally table for each result.
 When you have finished, fill in the frequencies.
 Check that the total frequency is correct.

- B2 Which had the greater frequency, home wins or away wins?

Booklet 4(a) (Grouping) (p.4)

- A4** Here are the frequencies for another herd of cows.
Draw a frequency chart to show them.

Weight in kg	Frequency
400 to 419	7
420 to 439	12
440 to 459	9
460 to 479	5
480 to 499	3

Booklet 4(b) (Comparing) (p.9)

- D5** The total yield of 38 Jersey cows was 127 642 litres in a year.
The total yield of 44 Guernsey cows was 156 508 litres.

- (a) What was the Jerseys' mean yield per cow in that year?
- (b) What was the Guernseys' mean yield per cow?
- (c) Which herd had the greater mean yield per cow?

Book Y2, Chapter 8 (p.73)

- C2** Here are the ages of the people on a coach outing.

9, 8, 9, 9, 10, 8, 10, 10, 9, 42, 8, 9, 9, 8, 8, 53, 8, 9, 9, 10, 10, 9

- (a) Does it make sense to calculate the mean age of the whole group?
If not, why not?
- (b) Calculate the mean age of the children in the group.

Discussion

It can be seen that Statistics was given much greater weight in England and Hong Kong than in China. The English textbooks contained a lot of examples and the emphasis was clearly on the interpretation of statistics graphs and on the applications of statistics. The emphasis in the Chinese textbook, on the other hand, was on calculation and manipulation of formulae. It seems that the topic of Statistics was treated as an application of Algebraic or arithmetic techniques. The Hong Kong textbooks discussed a lot of different statistical graphs and charts, and the emphasis was on providing students with a systematic and exhaustive list of different ways of presenting statistical data. So roughly speaking, in the study of Statistics, the Chinese textbook emphasized the manipulation of data, the Hong Kong textbooks emphasized the presentation of data while the English textbooks emphasized the interpretation of data.

Summary

From the brief analysis of the four topics taken from textbooks of the three places presented above, it can be seen that the mathematics content differed greatly from place to place. In Numbers and Algebra, the content in the Chinese textbooks was very abstract, and there seemed to be a preoccupation with generating rules. The content in the English textbooks was more concrete, and there was more reference to everyday life examples. The content in the Hong Kong textbooks was less abstract than that in the

Chinese textbooks but less concrete than that in the English ones, and there seemed to be a great emphasis on skills in calculation. In Geometry, the Chinese textbooks emphasized geometric proofs much more than those in Hong Kong and England. In Statistics, the Chinese textbooks emphasized the manipulation of statistical data, the Hong Kong textbooks emphasized the presentation of data, while the emphasis in the English textbooks was on the interpretation of data.

The analysis above also shows that textbooks in England intended to cater for students of different abilities through providing different mathematics content or mathematics content of different levels in different series of textbooks, while the textbooks in China and Hong Kong assumed that all students were to learn the same content. The mathematics content in the Chinese textbooks was in general more demanding, the approach of the text was more abstract and rigorous, and the treatment of content was more thorough. The treatment of mathematics content in Hong Kong and English textbooks, on the other hand, was more shallow, and the emphasis seemed to be on breadth rather than depth. The Hong Kong textbooks also seemed to stress practice of mathematical skills more than those in the other two places, while the treatment in the English textbooks was more concrete, and a variety of approaches were used.

As results from the classroom observations in this study (see chapter seven) show that the teaching in all three places was very much influenced by the textbooks used, the analysis presented above suggests that students in the three places were exposed to very different mathematical content and approaches to mathematics. Whether this is consistent with the intentions of the respective systems will be examined in the next chapter.

Chapter 9: Overall Discussion

In the last four chapters, evidence has been presented showing the differences in the intended curriculum, teachers' attitudes and the implemented curriculum in the three places (see chapter one, research questions 3 to 8). In this chapter, the two interpretative research questions (9 and 10) set out in chapter one will be addressed.

Firstly, the consistency of the intended curriculum and the implemented curriculum in the three places will be assessed. The intended curricula and the classroom practices in the three places concerned were compared separately in the last few chapters, but according to the model developed in chapter two, there should be a "congruent" relationship between these two components of the curriculum (see Stake, 1967:532). Whether this relationship was actually found in this study for the three places will be discussed below. Indeed the consistency between these two levels of the curriculum and the reasons for any inconsistencies are fundamental issues for curriculum developers, for all curriculum developers intend the planned curriculum to be implemented.

The second question focuses on explanations of differences in the mathematics curriculum, and in particular the differences in classroom practices in the three places. What factors can be suggested to account for the great differences presented in the past few chapters? In particular, if there were discrepancies between the intended curriculum and classroom practice and the former did not necessarily determine the latter, what then were the determinants of classroom practice? Again this is a vital question for both curriculum

developers and all those involved in education. By looking at the differences in classroom practice in the three places under study and trying to account for the differences using a variety of factors, it is hoped that light will be shed on what the possible determinants of classroom practice are.

Consistency of the Intended Curriculum and Implemented Curriculum

In contrast to the finding in the previous chapters that great between-country differences existed in the intended curricula, teachers' attitudes and the implemented curricula, it was found that there were elements of inconsistency between the intended and implemented curriculum in all three places, although these elements were not always the same for the three places. The consistency or inconsistency in aims, content and methods for each of the three places will be investigated in turn below.

Beijing

It was pointed out in chapter eight that the unified textbooks in China contained a lot of demanding mathematics content with rigorous discussion on mathematics concepts. It was also noted in chapter seven that Chinese teachers followed the textbooks closely in their lessons, and that they put great emphasis on explaining mathematics concepts in their teaching. So the aim of training of the mind stressed in the intended curriculum seemed to have been successfully implemented in the classroom.

The aim on the applications of mathematics might also have been accomplished through the inclusion of application problems in the unified textbooks, but as pointed out in chapter seven, a corresponding stress in the classroom teaching was not observed in this study. This may be reflection of a more "conservative" view of mathematics held by teachers than that held by the curriculum developers (or textbook writers), that is, whereas the curriculum developers saw mathematics as a subject relevant to everyday life, mathematics teachers seemed to view mathematics as a body of abstract knowledge to be learned in the classroom and which was unrelated to daily living. This conjecture is consistent with the finding on the attitudes of Chinese teachers towards mathematics expressed in the questionnaire (see chapter six).

Aims that were stressed in the intended curriculum but were not manifested in the classroom teaching included those on the fostering of good qualities of character and ideological education. It may be that these were achieved through more subtle means, or that these were mere rhetoric from the education authorities and were values not shared by the classroom teacher. The latter suggestion is understandable for teachers (note that more than 40% of the teachers in Beijing who responded to the questionnaire were over 45) who had gone through long periods of unsettled political movements and had now reached a period in the country of relative stability (see the description of the brief history in chapter four). It would be reasonable for them to want to concentrate on academic rather than ideological matters.

The intended use of a variety of teaching methods also did not materialize. The predominant method was teacher talk with involvement of students through questioning

and class-work. Discussions and activities (experiments) were rare. The large class size in Beijing schools which made group discussions and activities difficult to be carried out may be one reason for this inconsistency, or it may also be a reflection of teachers' inflexible views on mathematics and mathematics teaching and learning as shown up in the questionnaire results.

As far as the content of the curriculum is concerned, there was high consistency between intention and implementation because of both a centralized system (e.g. common textbook, unified public examinations) and tight monitoring by the education authorities.

Hong Kong

The intended emphasis on applications of mathematics was observed in only a minority of classrooms in Hong Kong, and the stress on mathematics as tools for other subjects was not specifically exhibited by teachers. So it seems that the emphasis in the intended curriculum on treating mathematics as a tool was not shared by the mathematics teachers.

The aim of introducing a sense of pattern and power of mathematics also did not show up at all in the classroom teaching. Mathematics was treated more as a set of skills and techniques to be mastered through ample practice. Motivation for learning seemed to be mainly extrinsic, doing well in examinations was the goal. As pointed out in chapter seven of this thesis, the influence of examinations on mathematics teaching was apparent in all three places, but for Hong Kong, in the absence of other aims such as training of the mind or enjoying mathematics, the influence of examinations became paramount. The

competitive and pragmatic spirit of the Hong Kong society might also have contributed to this stress by teachers and students on mastery of skills and techniques in preparation for examinations.

For the same reason, mathematics content was found to be the most consistent element between the intended and implemented curriculum in Hong Kong. What was taught in the classroom was largely determined by what was stipulated in the examination and teaching syllabuses. During interviews with the Hong Kong teachers, many of them complained that the syllabus was too heavy for the teaching time available, and they had to try hard to "cover the syllabus". So it is not surprising that departures in content from what was required in the syllabus were rarely observed in the Hong Kong classroom.

As far as teaching methods are concerned, again great discrepancy between intention and implementation was observed. Practical activities, whether conducted for the whole class, in small groups or individually, were rarely found in the classroom. The predominant scene in the Hong Kong classroom was that of the teacher lecturing and students practising on what was prescribed in the syllabus.

London

The intended curriculum in England contained a comprehensive list of aims. Since many lessons observed in London followed individualized learning programmes, and textbooks and other learning materials used (which provided nearly all the learning experiences for

students) reflected the intended aims quite well, consistency between intention and implementation can be considered to be high in this area. For example, the mathematics textbooks and learning materials in London contained many application problems and examples from everyday life as well as from other subject areas. The textbooks were usually colourfully produced with copious pictures and illustrations, and were seemingly intended to make the learning easier and more enjoyable for students. This is consistent with the observation that students in London seemed to enjoy their lessons more than their counter-parts in Hong Kong and Beijing. So the aims of applications of mathematics and enjoying mathematics seem to have been accomplished.

The mathematics taught in the London lessons was in general less demanding than that taught in corresponding Beijing and Hong Kong lessons, and for the general student population, this may help develop their confidence in mathematics as intended. The trade-off may have been less emphasis given to developing the "power of logical thinking" and "in-depth study of mathematics" (see chapter five).

For this study, the intended content was taken from the attainment targets in the National Curriculum. Since implementation of the National Curriculum had just started when the classroom observations in this study were conducted, many schools were still adjusting to the change and the textbooks used had not yet been updated to reflect the National Curriculum. So some discrepancy between the intended and implemented content was inevitable. During the interviews with teachers, however, they all indicated that adjustments had been or were going to be made to the teaching content to bring it in line with the National Curriculum.

As for teaching methods, the intended curriculum suggested a variety of modes and approaches of teaching. For those lessons that followed individualized programmes, however, the only mode of teaching and learning observed was seat-work with occasional discussion between students sitting next to each other. There was hardly any exposition by the teacher, and discussions between teacher and students were rare. From the analysis of the learning materials used, it was found that the seat-work included practical work, practising of skills and some investigation activities. Since students followed the individualized learning materials closely during those lessons, it could be considered that high consistency in learning approaches between intention and implementation for these areas had been achieved.

For lessons that did not follow individualized programmes, the teaching methods used were mainly exposition by the teacher, and students engaging in seat-work. Problem solving and investigations activities were also observed in a substantial number of classrooms. Intended methods that were not implemented included discussions and practical work.

Discussion

From the discussions above, it can be seen that many of the intended aims in Beijing and Hong Kong were not reflected in the way teachers taught in the classroom. It is possible that some of the aims were rhetorical, put in the intended curriculum documents because of political reasons and incorporating values not shared by the classroom teacher, or may be the practical day to day classroom constraints made the implementation of those aims

not viable. For London, the consistency between intention and implementation was higher. This happened because the textbooks and learning materials used were able to reflect the aims in the intended curriculum. The intended aims were "built into" the implementation through the textbooks and learning materials.

In contrast, the consistency between intention and implementation in the content of the curriculum was high in Beijing and Hong Kong (the case of London was unclear, see discussion above) because of the influence of public examinations in the two places, and because of the use of unified textbooks in China. So again it can be seen that high consistency was achieved because the intentions were "built into" the system.

The consistency between intention and implementation in teaching methods was higher in London than in the other two places, again because the intended methods were "built into" the system through the textbooks and learning materials used. For Beijing and Hong Kong, not only were some of the intended methods not "built into" the textbooks (the presentations in the textbooks were not consistent with intended teaching and learning approaches such as discussion and practical activities), but those methods were perceived by teachers as not effective for the purpose of preparation of students for examinations (the examinations did not require of students the experience of discussion and practical activities) and so were not implemented. This discrepancy may be the consequence of another mismatch between the values held by the curriculum developers and those held by classroom teachers, or the result of restrictions of classroom constraints, or simply a result of teachers' well-developed teaching habits which were robust against changes unless the changes were "built into" the system.

The discussion above seems to imply that there was in general a mismatch between the values embodied in the intended curriculum and those held by the classroom teacher, and that classroom practice was determined more by day to day classroom constraints rather than through teachers implementing the intended curriculum. Consistency between intention and implementation could only be achieved through "building" the intention into the system (e.g. by requiring schools to use certain scrutinized textbooks and through public examinations) rather than through convincing teachers of the system intention.

Two questions evolve from the discussions above. Firstly, for those classroom practices that were not consistent with the intended curriculum because the intentions were not "built into" the system, what were the most important determinants? Secondly, for those classroom practices that were consistent with the intended curriculum because of system stipulations, the question that still remains is why the system stipulations (or the lack of them) were different from place to place. These questions cannot be fully answered through resorting to curricular factors alone, and explanations have to be sought elsewhere. This is the task in the next section of this chapter.

A Framework for Explaining Differences in Curriculum

As pointed out in chapter three, one focus of the present study is the influence of culture on the mathematics curriculum, but it is also evident from the discussions in the last few chapters that the cultural explanation is not sufficient in explaining all the differences in

curriculum found in the three places, and other factors such as the constraints of classroom practicalities might also have contributed to the differences found. What is attempted in this section of the thesis is to find a comprehensive framework which encompasses all the factors that might have influenced the mathematics curriculum in the three places for the purpose of interpreting the findings in this study.

One such comprehensive model or structure was suggested by Bishop (1987, see also 1988b) when he was discussing the social dimension of mathematics education. Bishop suggested five levels in this dimension:

1. Cultural - the level of culture.
2. Societal - the level of society.
3. Institutional - the within-institution (school) level.
4. Pedagogical - the level of the classroom.
5. Individual - the level of the individual.

Since this study is focused on the intended and implemented curriculum rather than students' learning or achievement, the last level is not relevant to the present discussion. Bishop's use of these levels was for "conceptualising the social dimension of mathematics education" and to "offer a structure for the analysis and development of research in this area" (1987:3), and so the levels were defined in a general way. But with minor adaptations, the structure proved to be a useful framework for interpreting the findings in the present study. The definitions of the four levels used for the present study are given below:

Cultural level

The way the term "culture" is used in this study has been discussed in chapter one. "Culture " is a very broad and complicated term, and White divides the components of culture into four categories: ideological (composed of beliefs), sociological (i.e. customs, institutions, rules and patterns of interpersonal behaviour, etc.), sentimental or attitudinal and technological (1959:6-7, quoted in Bishop, 1988b:16). In the interpretation of the results in the present study, culture refers specifically to ideologies or values held by the group concerned (the first category in White's components of culture). In particular, the pertinent ideologies for this study are views on mathematics, views on education in general and on mathematics education in particular.

Societal level

In this study, the societal level refers to the structure and organization of the community, and in particular the political system and the economy. Of special relevance is the education system in the community.

Institutional level

In this study, institutions refer to the schools under study. Factors at the institutional level include policies and structures within the schools (e.g. time tabling) and the organization for mathematics teaching in particular. These are decisions made by individual schools rather than imposed by the system (the latter belong to the societal level).

Pedagogical level

Here the focus is on factors that affect teacher's decision in the teaching of mathematics in the classroom. It includes factors involving the teacher (e.g. values held by the teacher), the students (e.g. discipline) and the classroom setting (e.g. physical environment).

Explanation of Differences in the Mathematics Curriculum

Differences in the Intended Curriculum

The intended curriculum in this study is by definition intention at the system level. So in accounting for differences in the intended curriculum between the three places, factors at the cultural and societal levels in the framework suggested above will be drawn upon to offer explanation¹.

¹ It can be argued that the intended curriculum may be influenced by constraints at the institutional and classroom levels as well, but when consistency between the intended curriculum and the school and classroom practice is found, it would be difficult to decide whether the intended curriculum is influenced by school and classroom practice or the schools and classrooms are implementing the system intention. So explanation of differences in intended curriculum is not sought at the institutional and classroom levels here.

Great differences in aims of mathematics education and in mathematics content were found between the intended curriculum of the three places. For the aims of mathematics education, whereas the intended curriculum in the three places did not differ much in their aims on the utility of the product of mathematics, there were striking differences in their intrinsic aims and aims on the utility of the process of mathematics. Intrinsic aims were missing in the Chinese curriculum, while the Hong Kong curriculum did not have aims on the utility of the process of mathematics. The substance of the aims on the utility of the process of mathematics also differed between China and England (see chapter five).

In China, the aims in mathematics education were clearly very much influenced by the political mood of the country. The national goal of the country was the four modernizations, and all areas of education, including mathematics education, had to serve this national goal. And in a culture where the interests and rights of the individual are to submit to the interests of the community, it is not surprising to find that aims on contribution to the development of the country were specifically stated whereas aims directed at the development of the individual were de-emphasized.

In contrast, the aims on the utility of the process of mathematics in the intended curriculum in England emphasized the development of the individual, and this is consistent with the "individual orientation" of Western cultures (see chapter four). That aims on the utility of the process of mathematics were missing in the Hong Kong curriculum may be a reflection of the pragmatic attitude in the Hong Kong society: the Hong Kong people simply did not believe that the study of mathematics could contribute to the "fostering of good qualities of character".

The intended curricula in the three places were also found to differ tremendously in the mathematics content that students were supposed to learn. The Chinese put great emphasis on Algebra and Geometry, the curriculum in England stressed Numbers and Statistics, while the mathematics topics in Hong Kong curriculum were more uniformly distributed. The syllabus in China could be considered "narrow and deep", whereas the syllabuses in England and Hong Kong were relatively "wide and shallow".

These differences in mathematics content perhaps reflect different views on mathematics and mathematics education in the three places. The Chinese system was still very conservative towards what constituted mathematics and how mathematics content should be organized in the curriculum. Traditional subjects such as Algebra and Geometry were much more highly valued than relatively "new" subjects such as Statistics and Probability and the structure of mathematics was very much emphasized. When a certain topic was to be included in the mathematics curriculum, merely exposing students to a superficial notion of the topic was not enough. Topics should be either treated thoroughly or else excluded from the syllabus. So the resulting syllabus was not surprisingly "narrow and deep".

Mathematics, on the other hand, was viewed more flexibly in England. There was wide acceptance of relatively new topics such as Statistics and Probability into the curriculum, and it seemed that the process of mathematics was perceived as more important than the product of mathematics. So it was not necessary for topics to be treated thoroughly, for the important thing was for students to be exposed to the methods of mathematics rather than to learn a lot of mathematics results. There also seemed to be more attention given

to pedagogy rather than the structure of mathematics itself, and so topics which were judged to be too difficult for students were not included in the syllabus. This explains the "wide and shallow" syllabus in England.

Hong Kong seemed to be "mid way" between China and England in this aspect, and compared to China and England, the mathematics topics in the syllabus were more uniformly distributed. The mathematics content was more demanding than that in the English curriculum, but not as demanding as that in China, and its syllabus could still be regarded as "wide and shallow".

Differences in the Implemented Curriculum

Two salient differences in classroom practice observed in the three places were the organization of the mathematics teaching and the kind of instructional modes used. In Beijing and Hong Kong, teaching was predominantly done with the teacher expounding mathematical content in front of the whole class of students, and during class-work time, students were to practise the same skills just learned. In London, much more time was devoted to seat-work, and students usually learned mathematics individually through doing the tasks in the textbook. Also, different students were usually working on different tasks during seat-work time.

In explaining these differences at the pedagogical level of the framework suggested above, one can point to the limitations of large class size in Beijing and Hong Kong. In a class of 40 or more, it was difficult for the teacher to monitor the progress of all the students if they were doing different tasks and proceeding at different paces, and so the teacher had to resort to whole-class instruction.

This explanation can be related to explanations at higher levels in the framework. Policy on class size was sometimes an institutional decision, but eventually the limitation came from the financing of education at the system or societal level. Classes with less students were obviously more expensive to run than large classes, and since funds spent on education in China were so limited compared to the expenditure on education in England², the obvious solution for China was for teaching to be conducted in large classes if more children were to receive formal education.

These explanations at the lowest three levels of the framework encounter two difficulties. Firstly, they fail to explain the phenomenon of large class sizes in Hong Kong (and Japan and Taiwan), for although Hong Kong could afford to finance teaching in small classes, it did not choose to do so. For Hong Kong, the financial explanation is not adequate.

² The government expenditure in 1988 on education in the three places are as follows (State Education Commission (1991); Roberts (1989); Dennis (1991)):

	China	Hong Kong	United Kingdom
Government Education Expenditure as a percentage of GNP or GDP	2.3% (GNP)	2.6% (GDP)	4.4% (GDP)
Government Education Expenditure as a percentage of public expenditure	12.0%	17.6%	21.4%

GDP figures in China are not available, and so GNP figures are used in the table above.

Secondly, although expenditure on education was low in China, Chinese teachers enjoyed a much lighter teaching load than their counter-parts in Hong Kong and England. In China, the average mathematics teacher had to teach only two 45 minute lessons per day (Mondays to Saturdays, giving a total of 12 lessons per week, compared with the corresponding average figures of about 28 and 25 lessons in Hong Kong and London respectively), and the two lessons were usually "parallel" lessons on the same content so that the teacher only had to prepare for one lesson per day. Given the same amount of funding, smaller class size could have been achieved through increasing the workload of teachers³. But this did not happen in China.

More plausible explanations can be offered at the cultural or ideological level of the framework. Given that Chinese teachers viewed mathematics more as a product than a process (see chapter six), the most important thing for them in mathematics teaching was to have the mathematics content expounded clearly. So there was no need for smaller classes, because for lectures, class size has little effect on the efficiency of delivery of content (this argument is similar to the argument for large groups in lectures at university level). Moreover, if the assumption of the desirability of different students learning different things was not valid in Beijing, the problem of large class size did not exist at all, because whereas there were advocates of attending to individual differences in London, the predominant belief in China was that students should be taught the same thing in order to be fair to them.

³ There were other resource implications such as the availability of more teaching space for smaller classes, but these did not seem to be important factors in influencing decisions on class size.

In Hong Kong, the classroom practice was similar to that in Beijing, but there were some advocates of smaller class size from progressive educators. So Hong Kong can be considered as being in a state between Beijing and London, inclining more to the Beijing side, and this is consistent with the cultural relationship between the three places pointed out in chapter one.

It was not suggested above that explanations at the cultural level were the only explanations for the different practice in the three places, but the discussion above should highlight the inadequacy of explanations that ignored factors at the cultural level. The most probable cause for the differences was a combination of factors from all four levels.

Another finding that differentiated classrooms in the three places from each other was the amount of 'off-task' time and incidents of teacher maintaining discipline observed. 'Off-task' time included time when the teacher arrived at the classroom after the start-of-lesson bell, when teaching and learning stopped before the end-of-lesson bell, and when the teacher was engaged in administration or maintaining discipline while the class was not working on mathematics. As pointed out in chapter seven, these incidents happened more often in London schools when compared with schools in Hong Kong, which in turn had more 'off-task' time and incidents of teacher maintaining discipline than Beijing schools. It should be pointed out here that these differences might have immense effects on students' learning. Of course it was the quality of the teaching and learning activities which was of fundamental significance, but if the actual quantity of time available for mathematics learning differed significantly, the students who received less instructions (the London students in this case) might be at a disadvantage.

Obvious explanations of these differences are the different ways classes were organized in schools of the three places (institutional level) and the greater discipline problems in London schools (pedagogical or classroom level). In Beijing, students of the same class stayed in a fixed classroom. There was always a five or ten minute break between lessons, and teachers would usually arrive at the classroom before the bell and start the lesson as soon as the bell rang. In Hong Kong, for junior secondary classes, students of the same class usually stayed in a fixed classroom as well (though there were cases where students had to move from one classroom to another between lessons). But there might not be any "break" between lessons, and time had to be taken for the teacher to move from one classroom to another.

In London, mathematics lessons took place either in a special room or a room that was usually used by the particular teacher, and it was the students who were to move from one classroom to another and more time was needed for this moving. Also, since textbooks and all other learning materials were provided in maintained schools, time was needed for distributing and collecting these materials during lessons. On top of these demands, there was time spent on other administrative duties (such as taking roll call every lesson) and maintaining discipline by the teacher, and it was natural that this way of organisation would give rise to more 'off-task' time.

Two problems arise with this explanation. Firstly, if 'off-task' time was so undesirable, why were lessons in London organized in this way in the first place? There must be higher priority reasons for this way of organisation, and these reasons have to be sought

at the institutional and societal levels. Secondly, given that all three cities under study are metropolitan areas, why were there greater discipline problems in London? Again a cultural level explanation is suggested below.

The notion of 'off-task' time in this study is based on the assumption that all lesson time should be used for the teaching and learning of mathematics. This is indeed an assumption that seemed to be held by teachers in Beijing and Hong Kong. It was pointed out in chapter four that for the Chinese, learning is a serious endeavour, and one is not supposed to waste any time for learning. The classroom is the place for learning and students should not be engaged in irrelevant activities during lessons. In Hong Kong, on top of these Chinese cultural values held by most teachers, the cult of efficiency that is prevalent in the Hong Kong society might have also permeated into the classroom, and 'discipline' problems that got in the way of teaching were less tolerated by teachers. For London, it seems to the researcher that a different mentality existed in teachers. Study (and work) was to be enjoyed, and lesson time was interpreted flexibly. Digressions into something other than mathematics teaching and learning during lessons were tolerated, and it was not expected that the whole duration between the start-of-lesson and end-of-lesson bells be used for mathematics teaching and learning. So in the final analysis, what appeared to the researcher to be 'off-task' activities and discipline problems might not be considered problems at all for the London teacher. This was evident from the interviews with some of the teachers after observing their lessons. When interviewing teachers in London after observing what the researcher considered lessons with discipline problems, very often the interviewees would comment that they were satisfied with the discipline

of the class which they had just taught. But the typical reaction from Hong Kong teachers would be to apologise to the researcher for the bad behaviour of their students. (There was only one incident of a lesson with discipline problems observed in Beijing.)

Other major findings for the classroom observations include: Beijing lessons were highly structured while there were more extemporaneous instructional decisions made by the teachers in Hong Kong and London; London teachers seemed to stress learning mathematics through doing it while Hong Kong teachers stressed practising mathematical skills; there was more use of rigorous mathematical language in the Beijing classrooms in contrast to the non-technical language used in London; and the classroom practice of teachers in Beijing reflected a less flexible view of mathematics and mathematics education, a greater stress on conformity and a stronger emphasis on memorization than that of Hong Kong teachers, and in turn the London teachers.

The highly structured lessons in Beijing and the meticulous exposition of mathematics concepts by Beijing teachers show that they prepared for their teaching thoroughly, and this was possible only because of their relatively light teaching load mentioned earlier. This thorough preparation for lessons may be reflection of a feeling of incompetence on the part of the Chinese teachers (see qualifications of teachers who responded to the questionnaire in chapter six), or it may well be a tradition of what is expected of a teacher in China. Incidents of extemporaneous instructional decisions made by Hong Kong and London teachers in their classrooms might have caused them to appear to be less well prepared in their lessons, and this was consistent with the heavy workload they had in

teaching. Hong Kong teachers' stress on practising skills may be a reflection of the strong influence of public examinations on classroom teaching. But these explanations at the institutional and societal levels are not sufficient in accounting for the dominant positions of rigorous mathematical language, conformity and memorization in Beijing teachers' teaching. Reasons for these characteristics, it is proposed below, lie in the views held by Beijing teachers on mathematics and mathematics teaching and learning.

It was found in chapter six that teachers in Beijing held a more rigid view on mathematics than their counter-parts in Hong Kong and London. This is consistent with the finding from the classroom observations that the teaching in Beijing reflected a less flexible view of mathematics held by the Beijing teachers. Mathematics was perceived as a fixed body of knowledge to be learned, and if necessary memorized, through hard work. So the important thing in teaching was to have mathematics concepts expounded thoroughly and accurately, and this was best achieved through a carefully structured lesson and through using a rigorous mathematical language. The Chinese national characters of conformity and compliance mentioned in chapter four also help to explain the uniformity of teaching styles observed in Beijing and the Chinese teachers' practice of requiring their students to learn mathematics in a standard way.

The emphasis in London classrooms on learning mathematics through doing it and the use of non-technical language in talking about mathematics also point to a different underlying understanding of what mathematics and mathematics education are. For London teachers, mathematics was more a process than a product, and it was something that you learned through doing it rather than a fixed body of knowledge imparted by the teacher to the

students. Extemporaneous decisions made in the classroom were not perceived as lack of preparation, but as responding flexibly to the classroom situation. Rigorous mathematical language was thought to be unnecessary or even undesirable for students at this level.

For Hong Kong, it seems that the influence of both the Chinese and English views was present. But the most dominant view of mathematics and mathematics education seems to be that mathematics was perceived as an important school subject that would help students move up the education ladder. So the important thing was to help students to perform well in examinations, and the best way to prepare students for this was through giving them ample chance of drill and practice.

The discussions above are not meant to be verifications of the suggested explanations for the differences in curriculum found in the three places. As was pointed out in chapter one, the purpose of the study is to suggest factors to account for the findings. The verification or testing of these suggestions, if it can be done at all, is not within the scope of this study and may be an area for further research. Some suggestions for further research will be given in the concluding chapter that follows.

Chapter 10: Conclusion

In this chapter, the major findings of the study will be summarized and the implications of the findings discussed. Then the limitations of the study will be pointed out and some areas for further research will be suggested.

Summary of Major Findings

In chapter one, the research problem of the study was broken down into 10 questions. In this section, the findings of the study will be summarized by giving a brief answer to each of the 10 questions.

1. Can a model of a curriculum be developed based on which the mathematics curricula in the three places can be compared?

The model of a curriculum developed in chapter two conceives a curriculum as consisting of three levels: the intended, implemented and attained levels. The intended level comprises the three components aims, methods and content; the implemented level comprises methods and content; and the attained level consists of student achievement and attitudes. Teachers' attitudes mediate between the intended and implemented levels, while assessment procedures and instruments mediate between the implemented and attained levels. The model proved to be useful both for identifying the components of the

mathematics curriculum in the three places for comparison and for the consistency of the various components within the curriculum to be examined.

2. What are the contexts that give rise to the current mathematics curricula in the three places?

The three places under study were found to differ tremendously in their culture, education system, and history of development of the mathematics curriculum (chapter four). The Chinese culture, when compared to the West, stresses compliance, obedience, memorization and practice. Studying is considered a hardship, and there is a high expectation on students to achieve well. China is a huge country with a highly centralized school system, and resources in education are relatively scarce. England, in comparison, has a decentralized education system, and there are more resources allocated to education. Hong Kong is minute in scale compared with the other two places, and the education system is centralized.

The mathematics curriculum in China has gone through revolutionary changes in the past 40 years. The curriculum is very uniform throughout the country, though recently there is an intention to move towards more diversity. The mathematics curriculum in England underwent drastic changes in the sixties and seventies. The curriculum is very decentralized, but is currently moving towards more centralization. The mathematics curriculum in Hong Kong also underwent drastic changes in the sixties and seventies, but has now reached a stage of relative stagnation. The curriculum is centralized and uniform.

3. What are the differences in aims and objectives of mathematics education in the three places?

The curricula in the three places did not differ very much in their utilitarian aims on the product of mathematics (chapter five), all stressing the roles of mathematics in everyday life, in the overall curriculum, and for further study. The intended curriculum in Hong Kong did not include aims on the utility of the process of mathematics. The Chinese aims in this area stressed fostering of characters that contribute to the development of the country and ideological education, while the English aims laid more emphasis on the development of the individual. On the other hand, intrinsic aims were missing from the intended curriculum in China. Intrinsic aims in the Hong Kong and English curricula stressed appreciation of the pattern and power of mathematics, and realization of the role of mathematics in our culture; but treating mathematics as a source of delight and wonder was only found in the English aims.

4. What are the differences in intended teaching methods in the three places?

The intended curriculum in all three places proposed the use of a variety of teaching methods including lecturing, questions and answers, practising mathematical skills, and practical activities (chapter five). Investigations by students and discussions between teacher and students were only stressed in the curriculum in England, while the Chinese curriculum was the only one that mentioned competition between students.

5. What are the differences in intended mathematics content in the three places?

Great differences were found between the intended content of the curriculum in the three places (chapter five). The Chinese put great emphasis on Algebra and Geometry, while the English put more emphasis on Numbers and Probability and Statistics. The topics in the Hong Kong syllabus were more uniformly distributed.

6. What are the differences in teachers' attitudes towards mathematics and mathematics education in the three places?

Teachers' Attitudes towards Mathematics

ANOVA tests showed that teachers in the three places differed significantly in their attitudes towards mathematics (chapter six). Moreover, a pattern of responses to the questionnaire on teachers' attitudes towards mathematics emerged, with the responses of the Hong Kong teachers lying between those of the Beijing and London teachers. Teachers in Beijing tended to view mathematics as a rule-oriented and fixed discipline, while teachers in London perceived mathematics as more heuristic and changing.

Teachers' Views on Mathematics Education

Again ANOVA tests showed that teachers in the three places held very different views on mathematics education (chapter six). Major findings from the responses of teachers to the questionnaire included:

- a. Teachers in Beijing and London valued the subject mathematics more than their counterparts in Hong Kong.
- b. Teachers in Beijing thought that mathematics as tools for other subjects was the most important aim of mathematics education, teachers in Hong Kong thought that training of the mind was the most important, while teachers in London thought the ability to communicate logically and concisely most important.
- c. On factors that influenced the mathematics content taught in the classroom, the most important consideration for Hong Kong and London teachers was whether the content was interesting and meaningful for students, while for Beijing teachers, the syllabus was the most important.
- d. Beijing teachers believed that efforts paid by students and teachers were the most important factors that affected students' success and failure in mathematics, London teachers thought that the ability of students was the most important factor, while Hong Kong teachers thought that effort and ability were equally important.
- e. Beijing students were expected to spend more time on homework than their counterparts in Hong Kong, who in turn were expected to do much more homework than students in London.

The ANOVA tests also indicated that in general the differences in attitudes found above remained valid even when the diverse background characteristics of the teachers in the three places were taken into account.

7. What are the differences in methods of teaching in the three places?

From the classroom observations conducted in this study (chapter seven), it was found that group work was rare in all three places. Whole-class instruction was much more common in Beijing and Hong Kong than in London, whereas London students spent substantially more time doing "seat-work". It was also found that Beijing teachers laid more emphasis on mathematics concepts in their teaching, Hong Kong teachers laid more emphasis on practising mathematical skills, while London teachers laid more emphasis on discussions and learning mathematics through doing it. There were more "off-task" time and incidents of maintaining discipline observed in London than in Hong Kong, and in turn Beijing. The content of teaching in all three places was very much influenced by the textbook, but there was greater variety of content learned in London than in Hong Kong and Beijing.

The classroom practice of Beijing teachers was found to be much more uniform than those of Hong Kong and English teachers. The Beijing lessons were much more structured than Hong Kong lessons, and in turn the London lessons, while there were more incidents of extemporaneous decisions in Hong Kong and London. There was more use of rigorous mathematical language in the Beijing classrooms, and the classroom practice of teachers in Beijing reflected a less flexible view of mathematics, a greater

stress on conformity and a stronger emphasis on memorization than that of Hong Kong teachers, and in turn the London teachers.

8. What are the differences in the mathematics content taught in the three places?

An analysis of some of the topics in the textbooks in the three places (chapter eight) shows that the mathematics content taught in the Chinese classroom was very demanding and abstract. There was a great stress on Algebra and Geometry, and geometric proofs were emphasized. In England, there was comparatively a greater stress in Statistics, and the mathematics content was in general more concrete. The mathematics content taught in Hong Kong was less abstract than that in China but less concrete than that in England, and there seemed to be a great stress on skills in calculation.

9. Are the various components of the curricula in the three places consistent with each other?

Mismatch was found between the intended and the implemented curriculum in all three places (chapter nine). It seems that classroom practice was determined more by day to day classroom constraints rather than through teachers implementing the intended curriculum, and consistency between intention and implementation could only be achieved through "building" the intention into the system.

10. What factors can be suggested to account for the differences in curricula in the three places?

Factors at cultural, societal, school and classroom levels were suggested to account for the differences in the curriculum of the three places (chapter nine). It was argued that factors at the classroom, school and societal levels alone were not sufficient in accounting for the differences found in this study, and factors at the cultural level had to be drawn upon for a satisfactory explanation of the findings.

Implications of the Findings

Great differences in various components of the mathematics curriculum were found between the three places. As pointed out in the last chapter, these differences were not accounted for by referring to curricular factors and factors at the classroom level alone, and societal and cultural factors had to be drawn upon to explain these differences more fully. As argued in chapter two, past research on comparison of the mathematics curriculum tended to ignore the societal and cultural factors at work on the curriculum, and the findings of this study should help underpin the effect of societal practices and cultural values on the curriculum. The choice of the subject of mathematics for comparison has also shown that even for a supposedly "universal" subject such as

mathematics, culture does make a difference, thus ruling out any suggestion that mathematics education can be culture-free. The choice of the China-Hong Kong-England trio for comparison has served to illustrate the influence of societal and cultural factors on the curriculum well.

The findings of this study demonstrate clearly the complicated nature of the curriculum. Great differences were found in nearly all of the various components of the curriculum in the three places, and mismatch between the intended curriculum and implementation was found in all three places. That mismatch was found between intention and implementation is consistent with findings from much previous research, but the results of this study show that the kind of mismatch might differ from place to place, and cultural and societal as well as institutional and classroom factors might have contributed to these different kinds of mismatch.

The complicated nature of the curriculum should caution researchers on comparative curriculum studies. In comparing a certain component of the curriculum between countries, due consideration should be given to the interaction between that component and other components, and the societal and cultural contexts of the curriculum should be fully taken account of. It is when that particular component of the curriculum under comparison is examined in the context of all other components and of the societal and cultural background that the comparative results can be meaningfully interpreted.

In particular, results of international comparisons on mathematics achievement should be interpreted in the light of both the differences in the intended and implemented curricula

and the societal and cultural contexts of the places under study. What is attained by students in achievement tests means very little unless we know what the intentions of the respective systems are, how the mathematics are transacted in the classroom as well as the classroom, institutional, societal and cultural factors for the different classroom practice. The same attainment may mean very different things in different contexts, and to simply compare attainment or a simple matching of the intended content and attainment without due considerations of the complicated nature of the curriculum gives only very limited or even misleading information about the curriculum. To judge the merits of the systems based on such information or any extrapolations beyond the findings themselves can be deceptive. And it is not likely that interpretations of differences in attainment sought merely among curricular variables without reference to the societal and cultural contexts will yield satisfactory explanations.

The complicated nature of the curriculum and the great differences in curriculum found in this study also suggest that no simple methodology is sufficient to capture all the salient features of the curriculum in places of diverse societal and cultural backgrounds. Data from different components of the curriculum, because of the different nature of the phenomena, need to be studied employing different methodologies, and a purely quantitative or purely qualitative approach is unlikely to be appropriate. The multi-approach methodology employed in this study has proved to be an appropriate and useful methodology, and it is hoped that it has contributed to the literature a flexible and comprehensive methodology in studying the curricula in places with diverse societal and cultural backgrounds.

This study also brings the mathematics curriculum in China, which was little known outside the country, to the knowledge of the Western academic world. The comparison of the Chinese mathematics curriculum with those of Hong Kong and England and the analysis of its various components should help educators in China and outside the country to gain a deeper understanding of mathematics education in China. The interpretation of the findings of the comparative study should also help educators to see the Chinese mathematics curriculum in its societal and cultural contexts, an appreciation of which is essential for decision making concerning curriculum changes. As the curriculum in China is presently moving towards diversification in response to the challenge of popular education, the findings of the present study should provide an informed reference for curriculum developers.

For educators in Hong Kong, the present study should provide a detailed analysis of the mathematics curriculum in contrast with those of the two countries with whom she is specially related and in the light of her societal and cultural background. This information is particularly important, as was argued in chapter one, because of the coming of 1997 when the sovereignty of the Colony will be returned to China. Knowing the differences between her curriculum and that of China is essential in considering possible changes, and a knowledge of the influence of societal and cultural factors on the curriculum and an understanding of the differences between the Chinese and a Western curriculum (that of England) should help curriculum developers in Hong Kong when considering the direction for changes in the future.

Finally, as England moves towards further centralization through the introduction of a national curriculum and other measures, a comparison of her curriculum with those of highly centralized systems such as China and Hong Kong may throw light on the present status of her curriculum as well as the possible side-effects of centralization. Looking at the curriculum from societal and cultural perspectives also helps to understand the determinants of changes.

Limitations

Because of the nature of this study, the researcher was limited in resources and time. This meant that the study had to be kept to a small scale and a more comprehensive sampling of data for study was not possible. However, China and England are both large and diverse countries, and samples used in small scale studies such as this one were inevitably not representative of the respective populations. In this study, the teacher questionnaires were administered and the classroom observations were conducted only in Beijing and London for the cases of China and England. The researcher was well aware that these capital cities could not be taken as representative of other areas of the countries concerned, and so the results of these parts of the study should be interpreted as illustrative rather than representative of the situations in the two countries. Notwithstanding these limitations, the results of the questionnaire were found to be

remarkably consistent with the findings in the SIMS for the cases of England and Hong Kong (see discussion in chapter six), and so the samples can be considered fairly representative of the respective populations for a study of this scale.

For the time consuming and expensive activity of classroom observations in Beijing, Hong Kong and London, again the limitation of time and resources meant that the researcher had to balance the number of schools sampled and the number of lessons observed in each school. For this study, in order to sample a spectrum of schools for study, 18 schools in each city were visited, but the researcher had to limit himself to observing two lessons per school. The consequences of this limitation were that the consistency of classroom performance of individual teachers could not be examined, and that within school differences in practice could not be investigated. This created uncertainties when the practices in the 18 schools in each city were aggregated to represent the practice in the respective city, and thus weakened the validity of the results. But the alternative of visiting less schools in each place and observing more lessons in each school would have meant even more serious limitations on the generalizability of the results and would render the comparison between cities dubious.

The nature of this study also meant that only one researcher was involved in the collection of data. For the data on classroom observation, whereas this eliminated the problem of inter-observer consistency and hence achieved high internal reliability, it also meant that biases of the researcher were difficult to detect and thus external reliability was reduced. Collection of data by only one researcher also meant that simultaneous collection of data in the three places was not possible. Thus the data in this study were collected neither

at the same school year nor at the same period of a certain school year for the three places, and this practice is less than desirable for comparative studies.

The fact that the three places under study are far away from each other has made a more reactive approach in collecting data not possible. For the collection of qualitative data, it is desirable for new categories of data to be collected as new information unfolds itself during the data collection process. In this study, although a three-stage data collecting process (preliminary visits, pilot study and main study) was used, new information was still being uncovered during the main study, but it was not feasible to go around the three cities again and again whenever a fresh point arose. The result of this limitation was that even when seemingly significant findings were observed in a certain place under study at a late stage of the research, it was not possible to include them in the comparison because the corresponding data were not available in the other two places.

The limitation on time also meant that the researcher could not stay in some of the places under study for prolonged periods. Although sufficient time was deemed to have been spent in all three places for data collection, the researcher did not have the experience of prolonged submersion in the communities of China and England to enable him to understand thoroughly the cultures of these two places, and secondary sources instead of first hand experiences had to be relied on in understanding these. This handicapped the researcher when he attempted to draw on societal and cultural factors in the interpretation of the research findings.

The final limitations arose from the subjects under study and have already been mentioned in chapter three when the methodology of the study was discussed. These included the reservedness of the Beijing teachers and their unfamiliarity with educational research, the unstable situation in London because of the recent implementation of the National Curriculum with the consequence that some teachers were too busy to be interviewed, and the lack of published materials about the mathematics curriculum in China and Hong Kong. These limitations impeded the researcher in his effort to collect more reliable and valid data for the study.

Further Research

The present research can be extended in a number of ways. Firstly, since the three places under study were undergoing rather drastic changes in their curricula when this study was conducted, a follow up study near the end of this Century will throw valuable light on the determinants of curriculum changes. By that time, the curriculum for nine year universal education in China will have been developed and will have started to be implemented, Hong Kong will be a special administrative region under China and any changes in curriculum will have started to surface, and the National Curriculum in England will have been fully implemented for some years. Collecting the same kind of data as those collected in the present study and making comparisons with findings of this study will yield three illuminating case studies, and the results will contribute greatly to our knowledge on factors that affect curriculum changes and implementation.

The present research can be supplemented by adding in the attainment component of the curriculum for study. In the curriculum model developed in chapter two, student attainment is an important component of the curriculum, but because of practical constraints and the limitation of the scale of the study, this part of the curriculum was not included in the present study. A comparative study based on the full model of a curriculum developed in this study will give an even more comprehensive picture of the curricula in the three places.

It was pointed out in the last section that the classroom observations in the present study were conducted in a less than desirable scale and manner because of the limitation of time and resources. A possible extension of the present study is to conduct more indepth case studies in the classrooms of the three places, thus providing more detailed information on the classroom practices. Another possibility is to sample more schools in each city for study. This will enable more reliable data to be collected and hence improve the generalizability of the findings.

An obvious extension of the present study is for the questionnaire and the classroom observations to be extended to other areas of China and England, thus allowing us to conclude about teacher attitudes and classroom practices in these two countries instead of in the two capital cities only. The study can also be extended to include other levels than the junior secondary level to be studied. This will allow other factors such as the effects of selection and the influence of public examinations to be investigated.

Finally, the discussions in chapter nine suggested that the curriculum was influenced by a variety of factors at different levels. Further research with methodologies sensitive to these various factors can be designed to establish the relationship between these factors and the curriculum and to test the relative strengths of these factors on the curriculum. A possible research strategy to investigate this would be to include other countries with similar cultural backgrounds in the comparative study (e.g. Japan, Taiwan, the Chinese communities in North America, and European countries that share a similar culture with England), and through observing whether countries with similar cultural backgrounds but different societal structures have similar curricular intentions and classroom practices, the relationship between cultural and societal factors and the curriculum can be clarified.

Concluding Remarks

It is obvious that for a study of this nature, there are many limitations that affect the reliability of the research findings, and care should be taken in drawing inferences from the results. But it is hoped that despite all these limitations, the present study has contributed to the literature an example of a balanced methodology in curriculum studies in a cross-cultural setting, and that the findings have demonstrated that the curriculum is affected not only by factors at classroom, institutional and societal levels, but by cultural factors as well. The mathematics curriculum is not culture-free. Culture matters.

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Appendix A: List of People Interviewed

	Name	Position/Institution	Date of Interview
China	Professor CAO Cai-han	Associate Professor in Mathematics Education, Beijing Normal University	4 June 1988
	Professor SU Shi-dong	Vice-President, Guangdong College of Education	22 August 1988
	Professor ZHONG Shan-ji	Professor in Mathematics Education, Beijing Normal University	3 December 1988
	Mr. ZHANG Xiao-da	Chief editor (Mathematics), People's Education Press	3 December 1988
	Professor DING Er-sheng	Professor in Mathematics Education, Beijing Normal University	6 December 1988
Hong Kong	Mrs. Vivien LEUNG	Retired Mathematics Teacher, for many years Chairperson of Curriculum Development Subcommittee (Mathematics)	10 July 1990
	Mr. CHEUNG Pak Hong	Lecturer in Mathematics Education, University of Hong Kong	13 September 1990
	Mr. IP Chiu Kwan	Subject Officer (Mathematics), Hong Kong Examinations Authority	13 September 1990
	Mr. WONG Ngai Ying	Lecturer in Mathematics Education, Chinese University of Hong Kong	20 September 1990
	Mr. FUNG Tak Wah	Chairman, Curriculum Development Subcommittee (Mathematics)	25 September 1990

	Name	Position/Institution	Date of Interview
England	Mr. Chris CHRISTOFIDES	Mathematics Adviser, Borough of Enfield, London	20 March 1989
	Professor Celia HOYLES	Professor in Mathematics Education, Institute of Education, University of London	1 November 1990
	Dr. Margaret BROWN	Reader in Mathematics Education, King's College, University of London; Member of the National Curriculum Mathematics Working Group	17 April 1991

Appendix B: Questionnaire for Pilot Study

CODE: _____

Questionnaire for year one Mathematics Teachers in London

Section A

1. Sex: ☐ Male ☐ Female
2. Age: ☐ under 25 ☐ 25-34 ☐ 35-44
 ☐ 45-54 ☐ over 54
3. Qualifications
 - a) Academic
 - ☐ University graduate majoring in Mathematics
 - ☐ University graduate majoring in a subject other than Mathematics
 - ☐ Cert. of Ed.
 - ☐ A. Level
 - ☐ Others (Please specify: _____)
 - b) Professional
 - ☐ P.G.C.E.
 - ☐ M.Ed.
 - ☐ B.Ed.
 - ☐ No formal professional training
 - ☐ Others (Please specify: _____)
4. Teaching Experience (including the current year)
 - a) general: ☐ years
 - b) mathematics: ☐ years

5. Classes and subjects currently teaching:

Class	Subject	No of students	No of periods/week	No of minutes/period
1)				
2)				
3)				
4)				
5)				
6)				

6. Other duties in the school

- ☐ class teacher
- ☐ mathematics department head/panel chairman
- ☐ extra-curricular activities (Please specify: _____)
- ☐ others (Please specify: _____)

7. How much time do you use for each of the following activities per week?

- a) marking mathematics assignments and tests: _____ minutes
- b) preparing for mathematics lessons: _____ minutes
- c) responsibilities other than mathematics teaching: _____ min.

8. What is the average amount of time do you expect your year one students to spend on mathematics homework? _____ minutes/week

Section B

For questions 1 to 4, please rank order according to importance (1 stands for the most important, 2 stands for the second most important etc.):

1. Which do you think are the most important school subjects for year one students (Please rank order the 5 most important subjects):
☐ English ☐ Mathematics ☐ Science ☐ an European Language
☐ Humanities ☐ Geography ☐ History ☐ Art ☐ Music
☐ Physical Education ☐ Others (Please specify: _____)
2. Which of the below are the most important aims of mathematics education?
☐ application to everyday life
☐ tools for other subjects (e.g. Science)
☐ training of the mind
☐ appreciation of mathematics as part of our cultural heritage and for its intrinsic beauty
☐ ability to communicate logically and concisely
3. What are the most important factors affecting students' success or failure in mathematics?
☐ the ability of the student
☐ the effort paid by the student
☐ the teacher's teaching
☐ luck
☐ the support from the student's family

4. Which of the following should have the greatest influence on the mathematics content taught in the year one classroom?

- ☐ the syllabus
- ☐ the textbook(s)
- ☐ whether the content is interesting and meaningful for students
- ☐ whether the content helps to train students' minds or not
- ☐ whether the content is useful for students in their future or not

For questions 5 to 23, please indicate the extent of the agreement between the feeling expressed in each statement and your own personal feelings. (S.A. stands for strongly agree, A. stands for agree, N. stands for neutral, D. stands for disagree, and S.D. stands for strongly disagree)

	<u>S.A.</u>	<u>A.</u>	<u>N.</u>	<u>D.</u>	<u>S.D.</u>
5. Boys are better than girls in mathematics	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6. All children irrespective of their abilities can learn mathematics well	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
7. Students in the same class but of different abilities should learn different things in mathematics	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8. Students of roughly the same standard should be grouped together in sets for instruction	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
9. Mathematics will change rapidly in the near future	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	<u>S.A.</u>	<u>A.</u>	<u>N.</u>	<u>D.</u>	<u>S.D.</u>
10. Mathematics is a good field for creative people	—	—	—	—	—
11. There is little place for originality in solving mathematics problems	—	—	—	—	—
12. New discoveries in mathematics are constantly being made	—	—	—	—	—
13. Mathematics helps one to think according to strict rules	—	—	—	—	—
14. Estimating is an important mathematics skill	—	—	—	—	—
15. There are many different ways to solve most mathematics problems	—	—	—	—	—
16. Learning mathematics involves mostly memorizing	—	—	—	—	—
17. In mathematics, problems can be solved without using rules	—	—	—	—	—
18. Trial and error can often be used to solve a mathematics problem	—	—	—	—	—
19. There is always a rule to follow in solving a mathematics problem	—	—	—	—	—
20. There have not been any new discoveries in mathematics for a long time	—	—	—	—	—
21. Mathematics is a set of rules	—	—	—	—	—
22. A mathematics problem can always be solved in different ways	—	—	—	—	—
23. Mathematics helps one to think logically	—	—	—	—	—

Thank you very much for completing this questionnaire.

Appendix C: Questionnaire for the Main Study

I. Beijing

II. Hong Kong

III. London

初中数学教师问卷

甲 部

- 一) 性别 : ☐ 男 ☐ 女
- 二) 年龄 : ☐ 25岁以下 ☐ 25-34岁 ☐ 35-44岁
☐ 45-54岁 ☐ 54岁以上
- 三) 受教育程度: ☐ 高中毕业
☐ 中师毕业
☐ 中专毕业, 非中师
☐ 师专毕业
☐ 大专毕业, 非师专
☐ 普通大学毕业, 数学系
☐ 普通大学毕业, 非数学系
☐ 师范本科毕业, 数学系
☐ 师范本科毕业, 非数学系
☐ 其他 (请注明) _____

四) 教学经验 (包括本年度在内)

- A. 一般教学经验 _____ 年
B. 数学教学经验 _____ 年

五) 平均每周给予初中学生的家庭作业, 学生需要多少时间完成? _____ 分钟

乙 部

第一至第四题, 请按重要性在 () 内填上号数 ("1" 代表最重要, "2" 代表次重要, 如此类推)

一) 您认为对初中学生而言, 下列那些科目最重要 (请选五项, 然后按重要性填上号数):

- () 语文 () 外语 () 数学 () 历史 () 地理 () 自然科学
() 政治 () 体育 () 音乐 () 美术 () 劳动 (理、化、生)
() 其他 (请注明) _____

二) 您认为初中数学教育最重要的目的是什么? (请按重要性填上号数)

- () 日常应用 () 作为其他科目 (如科学) 的基础工具
() 思维训练 () 欣赏数学作为文化的一部份及数学的美
() 训练学生逻辑的和精确的表达能力
() 作为升学考试的重要科目

三) 您认为学生数学成绩的好坏, 何者影响最深? (请按重要性填上号数)

- () 学生的天资 () 学生的努力 () 老师的教学
() 家庭的支持和辅导

四) 您认为下列各项对数学教学内容的选择, 何者影响最大? (请按重要性填上号数)

- () 教学大纲 () 教材 () 内容是否有趣和有意义 () 考试大纲
() 内容能否训练学生思维 () 内容对学生将来是否有用

	<u>非常同意</u>	<u>同意</u>	<u>中立</u>	<u>不同意</u>	<u>非常不同意</u>
一) 一般而言，男孩子的数学比女孩子好	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
二) 所有学生不论天资如何都能够学好数学	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
三) 同一班内，能力不同的学生应该学习不同的数学内容	<input type="checkbox"/>	<input type="checkbox"/>	- <input type="checkbox"/> -	- <input type="checkbox"/> -	<input type="checkbox"/>
四) 程度相当的学生应该编到同一班内受教	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
五) 在最近的将来，数学将会演变得很快	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
六) 对于有创造力的人，数学是一个好的研究范畴	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
七) 很难有创新性的方法去解答数学问题	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
八) 数学上不断有新发现	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
九) 数学助人根据严谨的规律去思考	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十) “估计”是一项重要的数学技巧	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十一) 大多数的数学问题都可用多种不同的方法去解答	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十二) 学习数学大部份是靠记忆的	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十三) 在数学上，不利用规则也可以解决问题	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十四) 数学上的问题通常可以用“不断尝试改正错误”的方法解决	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十五) 解答一个数学问题，总可以依着一些规则作为根据	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十六) 有一段颇长的时间数学上大概都没有甚么新的发现	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十七) 数学是一组规则	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十八) 一个数学问题时常都能用不同的方法解答	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
十九) 数学可以助人逻辑地思考	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>



UNIVERSITY OF HONG KONG
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April, 1990

Dear Mathematics Teacher,

Re: Questionnaire for Junior Secondary
Mathematics Teachers in Hong Kong

Thank you for helping to complete this questionnaire which seeks to collect your views on mathematics and mathematics teaching. This is part of a research on a comparative study of the mathematics education (at Junior secondary level) in Hong Kong, China and England.

All information collected in this study will be treated as confidential. From experience, I know it takes only about 10 minutes to complete this questionnaire. Please return the completed questionnaire to the undersigned direct or through your colleague who is nominating P.C.E.d. in our Department. If you have any comments on the questionnaire, please feel free to write them at the appropriate parts of the questionnaire or in the box below. Should you have any queries, please give me a ring at 8592334.

Thank you again and I look forward to receiving your completed questionnaire.

Yours sincerely,

Frederick K. S. Leung
Lecturer in Mathematics Education

Comments

Questionnaire for Junior Secondary Mathematics Teachers in Hong Kong

香港初中數學教師問卷

Section A

甲 一般

- (1) Sex: ☐ Male ☐ Female
性別: ☐ 男 ☐ 女
- (2) Age: ☐ under 25 ☐ 25-34 ☐ 35-44 ☐ 45-54 ☐ over 54
年齡: ☐ 25歲以下 ☐ 25-34歲 ☐ 35-44歲 ☐ 45-54歲 ☐ 54歲以上
- (3) Qualifications: (Please put a "✓" in the appropriate box(es).)
受教程度: [請在適當項目之 ☐ 內填上 "✓" 號。(可選多項)]
- ☐ University graduate majoring in Mathematics
大學畢業 (數學系)
- ☐ University graduate majoring in a subject other than Mathematics
大學畢業 (非數學系)
- ☐ Graduate of other tertiary institutions (e.g. Polytechnic, Baptist)
其他大專院校畢業 (如理工、浸會)
- ☐ Hong Kong A Level
香港高級程度考試
- ☐ Hong Kong B Level
香港中級程度考試
- ☐ HKCEE Level
香港中學程度考試
- ☐ P.C.E.d. Cert. Ed. or Dip.Ed.
學位教育證書
- ☐ M.Ed. 教育學碩士
- ☐ Ed. 學士
- ☐ Cert. Ed. [For a College of Education]
教育學院非學位教育文憑
- ☐ Informal professional training
非正式文憑培訓
- ☐ Others (Please specify): _____
其他 (請註明): _____

- (4) Teaching Experience (including the current year)
教學經驗 (包括本年度在內)

General: _____ years
一般教學經驗: _____ 年

Mathematics: _____ years
數學教學經驗: _____ 年

- (5) What is the average amount of time do you expect your Junior secondary students to spend on mathematics homework?
平均給予初中學生回家作業, 學生需要多少時間完成? 每週 _____ minutes/week
分鐘

Section B

2. 座

For questions 1 to 4, please rank order according to importance (1 stands for the most important, 2 stands for the second most important, etc.);
第1至第4題，請依重要性在 座 內填上編號 ("1" 代表最重要，"2" 代表次重要，如此類推)：

(1) Which do you think are the most important school subjects for junior secondary students? (Please rank order the 5 most important subjects);
如認為初中最重要科目最重要，下列科目填上編號 (請按重要性填上編號)：

Chinese	Integrated Science	Social Studies
國文	綜合科學	社會教育
English	Physical Education	Mathematics
英語	體育	數學
Music	Chinese History	Arts
音樂	中國歷史	藝術
Others (Please specify):		
其他 (請註明)：		

(2) What do you think are the most important aims of junior secondary mathematics education? (Please rank order);
如認為初中數學教育最重要的目的是什麼？ (請按重要性填上編號)

Application to everyday life	
日常應用	
Tools for other subjects (e.g. Science)	
作為其他科目 (如科學) 的基本工具	
Training of the mind	
訓練訓練	
Appreciation of mathematics as part of our cultural heritage and for its intrinsic worth	
欣賞數學作為文化的一部份及數學的奧妙	
Ability to communicate logically and concisely	
訓練學生邏輯的和簡明的表達能力	

(3) What do you think are the most important factors affecting students' success or failure in mathematics? (Please rank order);
如認為學生數學成績的好壞，何者影響最大？ (請按重要性填上編號)

the ability of the student	
學生的天資	
the effort paid by the student	
學生的努力	
the teacher's teaching	
老師的教學	
luck	
運氣	
the support from the student's family	
家庭的扶持和輔導	

(4) Which of the following do you think SHOULD have the greatest influence on the mathematics content taught in the classroom? (Please rank order);
如認為下列各項對數學教學內容的影響，何者影響最大？ (請按重要性填上編號)

the syllabus	
教學大綱	
the textbook(s)	
教科書	
whether the content is interesting and meaningful for students	
內容是否有趣和有意義	
whether the content helps to train students' minds	
內容是否訓練學生思維	
whether the content is useful for students in their future	
內容對學生將來是否有用	

Section C

For questions 1 to 10, please indicate by using a "✓" the extent of the agreement between the feeling expressed in each statement and your own personal feelings.
(S.A. stands for strongly agree, A. stands for agree, M. stands for neutral, D. stands for disagree, and S.O. stands for strongly disagree)
第1題至第10題，請以 "✓" 表達你對每句開意的程度：

非常同意
同意
中立
不同意
非常不同意

- (1) In general boys are better than girls in mathematics
一般而言，男孩子的數學比女孩子好
- (2) All students irrespective of their abilities can learn mathematics well
所有學生不論天資如何都能學好數學
- (3) Students in the same class but of different abilities should learn different things in mathematics
同一班內，能力不同的學生應學習不同的數學內容
- (4) Students of roughly the same standard should be grouped together in sets for instruction
程度相當的學生應被編組同一班內受教
- (5) Mathematics will change rapidly in the near future
在最近的將來，數學將會演變得很快
- (6) Mathematics is a good field for creative people
對於有創造力的人，數學是一個好的研究領域
- (7) There is little place for originality in solving mathematics problems
做數學問題時原創性方法去解答數學問題
- (8) New discoveries in mathematics are constantly being made
數學上不斷有新發現
- (9) Mathematics helps one to think according to strict rules
數學有助於人依照嚴格的規條去思考
- (10) Estimating is an important mathematics skill
"估計" 是一項重要的數學技巧
- (11) There are many different ways to solve most mathematics problems
大多數的數學問題都可用多種不同的方法去解答
- (12) Learning mathematics involves mostly memorizing
學習數學大部分是靠記憶的
- (13) In mathematics, problems can be solved without using rules
在數學上，不利用規則也可以解決問題
- (14) Trial and error can often be used to solve a mathematics problem
數學上的問題通常可以用 "不斷嘗試修正錯誤" 的方法解決
- (15) There is always a rule to follow in solving a mathematics problem
所有一個數學問題，總可以依循一些規則作為解答
- (16) There have not been any new discoveries in mathematics for a long time
有一段很長的時間數學上大概都沒有甚麼新的發現
- (17) Mathematics is a set of rules
數學是一堆規則
- (18) A mathematics problem can always be solved in different ways
一個數學問題通常都能用不同的方法解答
- (19) Mathematics helps one to think logically
數學可以助人邏輯地思考

Thank you very much for completing this questionnaire.
調查完 乞 勿 忙 忙

**INSTITUTE
OF EDUCATION**
UNIVERSITY OF LONDON

20 Bedford Way
London WC1H 0AL



December 1990

Dear Mathematics Teacher,

Re: Questionnaire for Junior Secondary
Mathematics Teachers in London

I am a lecturer in mathematics education from the University of Hong Kong currently working as a research student at the Institute of Education, University of London and engaged in a comparative study of mathematics education (at junior secondary level) in England, China and Hong Kong. I would like to invite you to help in completing this questionnaire which seeks to collect your views on mathematics and mathematics teaching.

All information collected in this study will be treated as CONFIDENTIAL. You may notice that there is a serial number on the top left hand corner of this page. This is for the purpose of following up those who do not return the questionnaire and is not for identifying individuals' views expressed in the questionnaire.

From experience, I know it takes only about 10 minutes to complete this questionnaire. Please return the completed questionnaire to the undersigned using the stamped envelope provided. Should you have any queries, please give me a ring at (071)-837-8888 ext. 4921.

Thank you very much for your help and I look forward to receiving your completed questionnaire.

Yours sincerely,

John

Fredrick Leung
Ph.D. student at the
Institute of Education,

Correspondence Address:
Flat 921
William Goodenough House
Mucklenburgh Square
London WC1N 2AN

Questionnaire for Junior Secondary Mathematics Teachers in London

Section A

1. Sex: ☐ Male ☐ Female

2. Age: ☐ under 25 ☐ 25-34 ☐ 35-44
☐ 45-54 ☐ over 54

3. Qualifications (you may tick more than one option)

<input type="checkbox"/>	University graduate majoring in Mathematics
<input type="checkbox"/>	University graduate majoring in a subject other than Mathematics
<input type="checkbox"/>	A-Level
<input type="checkbox"/>	P.G.C.E.
<input type="checkbox"/>	M.Ed.
<input type="checkbox"/>	B.Ed. (main subject: Mathematics)
<input type="checkbox"/>	B.Ed. (main subject not Mathematics)
<input type="checkbox"/>	Cert. of Ed. (main subject: Mathematics)
<input type="checkbox"/>	Cert. of Ed. (main subject not Mathematics)
<input type="checkbox"/>	No formal teacher training
<input type="checkbox"/>	Others (Please specify: _____)

4. Teaching Experience (including the current year)

general: _____ years

mathematics: _____ years

5. What is the average amount of time do you expect your junior secondary students to spend on mathematics homework?

_____ minutes/week

(P.T.O.)

Section C

For questions 1 to 19, please indicate by using a "✓" the extent of the agreement between the feeling expressed in each statement and your own personal feelings. (S.A. stands for strongly agree, A. stands for agree, N. stands for neutral, D. stands for disagree, and S.D. stands for strongly disagree.)

- S. A. A. N. D. S. D.

<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
English	Science	Humanities			
an European Language	Physical Education	Mathematics			
Music	History	Art			
Others (please specify): _____)					

1. In general boys are better than girls in mathematics
2. All students irrespective of their abilities can learn mathematics well
3. Students in the same class but of different abilities should learn different things in mathematics
4. Students of roughly the same standard should be grouped together in sets for instruction
5. Mathematics will change rapidly in the near future
6. Mathematics is a good field for creative people
7. There is little place for originality in solving mathematics problems
8. New discoveries in mathematics are constantly being made
9. Mathematics helps one to think according to strict rules
10. Estimating is an important mathematics skill
11. There are many different ways to solve most mathematics problems
12. Learning mathematics involves mostly memorizing
13. In mathematics, problems can be solved without using rules
14. Trial and error can often be used to solve a mathematics problem
15. There is always a rule to follow in solving a mathematics problem
16. There has not been any new discoveries in mathematics for a long time.
17. Mathematics is a set of rules
18. A mathematics problem can always be solved in different ways
19. Mathematics helps one to think logically

[illegible]

9. Mathematics helps one to think according to strict rules
10. Estimating is an important mathematics skill
11. There are many different ways to solve most mathematics problems
12. Learning mathematics involves mostly memorizing
13. In mathematics, problems can be solved without using rules
14. Trial and error can often be used to solve a mathematics problem
15. There is always a rule to follow in solving a mathematics problem
16. There has not been any new discoveries in mathematics for a long time.
17. Mathematics is a set of rules
18. A mathematics problem can always be solved in different ways
19. Mathematics helps one to think logically

[illegible]

- [illegible]

Thank you very much for completing this questionnaire.

Appendix D: Classroom Observation Schedule for the Pilot Study

CLASSROOM OBSERVATION

Code: _____
 School: _____ Date: _____
 Teacher: _____ Form: _____ Students: M _____ F _____
 Time: Start: _____ End: _____ Topic: _____

<u>Discipline</u>	<u>Instructions</u>	<u>Lecture^a</u>	<u>Questions^b</u>	<u>Cl Work^c</u>	<u>Gp Work^c</u>	<u>Silence</u>	<u>Others</u>
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-

A.1 Alternate Method

- 2 Own Method
- 3 Trial & Error
- 4 Memory
- 5 Rigor
- 6 Application
- 7 Beauty/Culture
- 8 Examination

B.1 To the whole class

- 2 To individuals M
F

C. Differential Treatment:

D. A-V aids

E. Homework:

F. Others:

Appendix E: Classroom Observation Schedule for the Main Study

CLASSROOM OBSERVATION

CODE: _____

School: _____ Date: _____

Teacher: _____ Form: _____ Students: M _____ F _____

Time: Start: _____ End: _____ Topic: _____

Questioning: M _____ High _____ Low _____
F _____

Boardwork: M _____
F _____

Organisation:

Time Description

W o n b g a
G i d a
I s d r a
X a o

W o n b g a
G i d a
I s d r a
X a o

W o n b g a
G i d a
I s d r a
X a o

W o n b g a
G i d a
I s d r a
X a o

W o n b g a
G i d a
I s d r a
X a o

W o n b g a
G i d a
I s d r a
X a o

Appendix F: Interview Schedule (for Teachers)

code:_____

School:_____Teacher:_____(M/F) Date:_____

1. For how long have you been a teacher?
2. Do you teach any subjects other than mathematics?
3. Is the lesson you just taught typical of the way you teach?
4. How do you select what content to cover and what methods of teaching to use?
5. What do you think is the place of lecturing/group work/activities in mathematics lessons?
6. What is the role of textbooks in your teaching?
7. Please comment on the mathematics textbooks that your school is using.
8. Please comment on the (official) mathematics syllabus.
9. About how much time per week do you use for preparing your lessons?
10. How do you find the workload as a teacher?
11. What is the most important thing that students should learn through mathematics lessons?
12. What do you think the aims of mathematics education should be for junior secondary school students?
13. What do you think is the role of teachers in the teaching and learning of mathematics?
14. Do you think all students should be able to get a pass in their mathematics tests and examinations?
15. What are the most important factors that contribute to students' success and failure in mathematics?
16. Do you think students should be grouped according to ability for mathematics instruction?
17. What are the most common problems that you as a mathematics teacher are encountering?
18. What are the most pressing problems of concern in mathematics education in your country?

Appendix G: Juxtaposition of the Content in the Three Curricula:

China	Hong Kong	England
Number	Numbers and Counting	Whole numbers and the Four Rules
		Decimals, Fractions
	Percentages	Percentages
	Rate, Ratio and Proportion	Unitary ratios
Negative Numbers and Rational Numbers	Negative Numbers	Negative Numbers
4 Rules of Rational Numbers		Trial and Improvement Methods
Approximation and Significant Figures	Approximation and Measurement (including significant figures)	Estimation and approximation (including significant figures)
	(Use of Calculators)	Use of Calculators
Tables of Squares and Cubes		Primes, Multiples, Factors
Square and Cube Roots		Square and Cube Roots
		Powers and Roots
Irrational Numbers and Real Numbers		
[Common Logarithm]	Common Logarithm	
Measures		(Scale in Maps and Drawings)
		Imperial and Metric Units
		Compound Measures (e.g. speed, density)

	Area and Volume (Circles, Rectangular Blocks, Prisms, Cylinders, Pyramids, Cones, Spheres; Similar Solids)	Perimeter, Area and Volume (Rectangles Triangles, //grams Trapezia, Cubes, Cuboids, Cylinders, Prisms and solids of constant cross area)
	Approximation and Measurement	Error in Measurement
Algebra		Sequence of Numbers, Number Pattern
	Formulae, Open Sentences	Formulae, Functions
Algebraic Expressions	Algebraic Expressions	Algebraic Expressions
Polynomials (4 Rules, Factorization)	Polynomials (4 Rules, Factorization)	[Polynomials?]
Rational Expressions		
Linear Equations in One Unknown	Linear Equations in One Unknown	Equations?
Linear Equations in Two Unknowns	Linear Equations in Two Unknowns	Simultaneous Linear Equations
Linear Equations in Three Unknowns		
Rational Equations reducible to linear equations		
	Equations and Identities	
Quadractic Surds		
Quadratic Equations	Quadratic Equations	
Rational and Irrational Equations reducible to quadratic equations		
System of Quadratic Equations		
		Trial and Improvement Methods for the Solution of Linear and Simple Polynomial Equations

Zero and Negative Indices [Fractional Indices and Surds]	Laws of Indices (including negative and fractional indices)	Rules of Indices (integral, negative and fractional indices)
Linear Inequalities in One Unknown	Linear Inequalities in One Unknown	Linear Inequalities in One Unknown Linear Inequalities in Two Unknowns
Functions		
Graphs of Linear and Quadratic Functions	[Graphs of linear and Quadratic Functions]	Graphs of Linear, Quadratic and Reciprocal Functions The role of counter-example in disproving a rule

Geometry

Geometric Bodies, Geometric Figures	Use of Protractor and Compasses	Construction of 2-D and 3-D Shapes
Points, Lines and Planes		
Relationship between Lines and Planes		
Line Segments, Rays Size and Midpoint of Segments	Line Segment Bisection	
Angle, Angle Measure	Properties of Angles and Simple Shapes	Measurement of Angle
Angle Bisection	Angle Bisection	
Classification of Angles, Vertically Opposite Angles, Adjacent Angles on a Straight Line, Supplementary Angles	Angles and Parallel Lines (Vertically Opposite Angles, Adjacent Angles on a Straight Line,	Properties Associated with Intersecting and Parallel Lines
Perpendicular Lines, Distance of a Point from a Line		

Corresponding Angles, Alternate Angles, Interior Angles	Corresponding Angles, Alternate Angles, Interior Angles)
---	--

Properties and
Determination of
Parallel Lines

Triangles (Angle Bisectors, Medians Altitudes of Triangles; Triangular Inequality; Angle Sum; Classification)	Angles of Triangles	Properties Associated with Triangles Angle and Symmetry Properties of Triangles
--	---------------------	---

Congruent Triangles: Properties and Determination	Conditions for Congruent Triangles	Congruence of Simple Shapes
---	---------------------------------------	--------------------------------

Isosceles Triangles,
Equilateral Triangles,
Right Angle Triangles:
Properties and
Determination

Pythagoras' Theorem and its Converse	Pythagoras' Theorem	Pythagoras' Theorem
---	---------------------	---------------------

Properties of Angle
Bisectors and
Perpendicular Bisectors

Locus; Axis of Symmetry, Symmetric Figures and Their Properties	Rotational Symmetry, Symmetries of Various Shapes; Locus of an Object Moving Subject to a Rule
---	--

Basic Geometric
Construction

Internal and External Angle Sum of Polygons	Angles of Polygons	Angles and Symmetry Properties of Quadrilaterals and Polygons
---	--------------------	--

Parallelogram,
Rectangle, Rhombus
Square, Trapezium:
Properties and
Determination

Classification of Quadrilaterals	Classification of Quadrilaterals
-------------------------------------	-------------------------------------

Area of Irregular
Polygons

Mid-point Theorem and Intercept Theorem	Mid-point Theorem and Intercept Theorem	
Similar Triangles, Similar Right-Angled Triangles: Properties and Determination	Similar Triangles Ratio of Areas of Similar Figures	Mathematical Similarity
Similar Polygons and Their Properties		
Projections and Measurement		
Circles, Relation between a Circle and a Straight Line, Relation between Circles, Regular Polygons and Circles		
		Networks, Traversability
		2-D representation of 3-D Objects
		Enlargement
		Vector Notation
Co-ordinate Geometry		
Plane Rectangular Co-ordinate System	Introduction to Co-ordinates	Co-ordinate Representation of points
	Use of Rectangular and Polar Co-ordinates	Location on Maps Specified by means of Co-ordinates
[Distance Formula]	Distance Formula, Gradient, Section Formula	
	Different Standard Forms of Straight Lines	Cartesian Co-ordinates to Represent Simple Function Mappings (Linear, Quadratic)

Trigonometry

Trigonometric Functions of Acute Angles (Sine, Cosine, Tangent and Cotangent)	Sine, Cosine and Tangent Ratios for Angles between 0 and 90 Degrees	Sine, Cosine and Tangent in 2-D
Special Angles (30, 45 and 60 Degrees)	Special Angles (30, 45 and 60 Degrees)	
Use of Trigonometric Tables	Use of Trigonometric Tables	
Solution of Right-angled Triangles	Solution of Right-angled Triangles	
Trigonometric Relations: $\sin a / \cos a = \tan a$ $\sin a + \cos a = 1$ $\sin(90-a) = \cos a$ $\cos(90-a) = \sin a$ $\tan(90-a) = \cot a$ $\cot(90-a) = \tan a$	Trigonometric Relations: $\sin a / \cos a = \tan a$ $\sin a + \cos a = 1$ $\sin(90-a) = \cos a$ $\cos(90-a) = \sin a$ $\tan(90-a) = 1/\tan a$	
Applications	Applications	

Statistics

Population and Sample	Collection of Data	Data Collection
Mean	Mean, Median, Mode	Mean, Median
Variance, Standard Deviation		Range, Interquartile Range
		Correlation
	Discrete Data, Grouped Data	Grouped Discrete Data, Grouped Continuous Data
Frequency Table	Frequency Table	Frequency Table
Histogram	Bar Chart, Pictogram, Pie Chart, Histogram, Frequency Polygon, Frequency Curve	Bar Chart, Line Graph, Pie Chart, Conversion Graphs, Scatter Graph Frequency Polygon, Cumulative Frequency Table and Curve, Histogram

Survey, Questionnaire

Hypothesis Testing

Computer Database

Flow Diagrams

Interpretation of
Statistical Graphs

Uses and Abuses of
Statistics

Probability

Terminology

Meaning of
Probability

Probability Scale

Experimental and
Theoretical
Probability

Experimental and
Theoretical
Probability

Independent Events,
Exclusive Events,
Addition Rule

Relative Frequency as
an Estimate of
Probability

Appendix H: Subtraction of Negative Numbers

China

Subtraction was introduced as the reverse of addition, and the following examples were discussed in introducing the subtraction of negative numbers (Algebra 1, p.25):

(1) $(+3) - (+5)$.

This expression means to find which number when added to +5 gives +3.

$$\because (+5) + (-2) = +3,$$

$$\therefore (+3) - (+5) = -2.$$

From the addition of rational numbers, we know that

$$(+3) + (-5) = -2.$$

$$\therefore (+3) - (+5) = (+3) + (-5).$$

(2) $(+3) - (-5)$.

This expression means to find which number when added to -5 gives +3.

$$\because (-5) + (+8) = +3,$$

$$\therefore (+3) - (-5) = +8.$$

From the addition of rational numbers, we know that

$$(+3) + (+5) = +8.$$

$$\therefore (+3) - (-5) = (+3) + (+5).$$

To sum up, we get the following rule for the subtraction of negative numbers:

"To subtract a number is the same as to add the (additive) inverse of the number"

Hong Kong

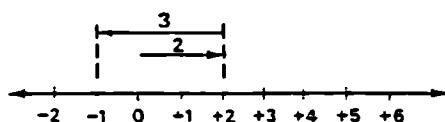
Like the treatment in the Chinese text, subtraction of directed numbers was introduced as the reverse of addition, but the discussion was done using the number line with the following examples (Book 1, p.222):



Example 2 With the help of a number line, find

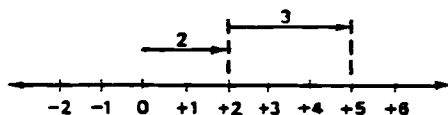
- (a) $(+2) - (+3)$
- (b) $(+2) - (-3)$

Solution (a) [Since subtraction is the reverse of addition, $(+2) - (+3)$ means that starting from 0, we move 2 units to the right and then 3 units to the left.]



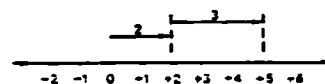
$$\therefore (+2) - (+3) = \underline{-1}$$

- (b) [Again since subtraction is the reverse of addition, $(+2) - (-3)$ means that starting from 0, we move 2 units to the right and then 3 units again to the right.]

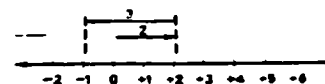


$$\therefore (+2) - (-3) = \underline{+5}$$

Recall:
The addition of directed numbers
 $(+2) + (+3)$ means



Recall:
The addition of directed numbers
 $(+2) + (-3)$ means



England

Subtraction of negative numbers was introduced through observing a pattern (R1, 3(e) and Y1) and/or using an example in a concrete situation (B1, R1 and 3(e)). The concept of subtraction of positive numbers was assumed and not discussed specifically, because unlike the cases in China and Hong Kong, the concept was not used for introducing subtraction of negative numbers. The treatment in booklet 3(e) is given below (p.9-12):

C1 Write the next four lines of each of these patterns.

(a) $3 - 2 = 1$

$$3 - 1 = 2$$

$$3 - 0 = 3$$

$$3 - -1 = 4$$

(b) $6 - 5 = 1$

$$6 - 4 = 2$$

$$6 - 3 = 3$$

$$6 - 2 = 4$$

(c) $10 - 4 = 6$

$$10 - 3 = 7$$

$$10 - 2 = 8$$

$$10 - 1 = 9$$

Here is the answer to
part (c) of question C1.

Compare it with this.

$$10 - 0 = 10$$

$$10 - -1 = 11$$

$$10 - -2 = 12$$

$$10 - -3 = 13$$

$$10 + 0 = 10$$

$$10 + 1 = 11$$

$$10 + 2 = 12$$

$$10 + 3 = 13$$

It looks as if subtracting -1 is the same as adding 1,

subtracting -2 is the same as adding 2,

subtracting -3 is the same as adding 3,

and so on.

The next question will help you to understand this.

C2 The pupils at Dotheboys School get marks for behaviour.

Very good 2, Good 1, Bad -1, Very bad -2

Here is Neil's record card for March.

	Name	Neil
(a) Check the total is correct.	Date	Marks
	March 5th	4
	March 9th	5
On March 31st Neil got -2	March 10th	3
for writing on the blackboard.	March 16th	-4
Later on, another boy owned up.	March 23rd	1
So the teacher took the -2	March 31st	-2
from Neil's marks.	Total	7

(b) Total up Neil's marks again but leave out the -2.

**Before the other boy
owned up, Neil had 7.**

**The teacher took
away -2.**

Now Neil has 9

$$7 - -2 = 9$$

You get the same answer if you do $7 + 2$ instead of $7 - -2$

Here again are the rules for adding and subtracting a negative number.

Adding -3 is the same as subtracting 3.

Subtracting -3 is the same as adding 3.